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ABSTRACT

This report presents an analysis of the content validity of a series of secondary school mathematics tests designed by university mathematicians in California to help improve the articulation between secondary and postsecondary mathematics instruction. The purpose in conducting this analysis was to improve understanding of what it means to validate a subject matter test that is designed to serve several purposes. The analysis itself serves as an early prototype in work on developing models for validating such tests. Thus, while the actual examination was specific to the tests under consideration, both the methods employed in the analysis and the issues raised in conducting them are intended to serve as guidance to other educational reform and improvement efforts that place heavy reliance on content assessments. Approximately one half of the report consists of 13 tables outlining analysis results, 6 pages of references, and 5 appendices composed of curriculum expectation statements in several publications and a content specification. (PK)

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ESTABLISHING THE CONTENT VALIDITY OF TESTS DESIGNED TO SERVE MULTIPLE PURPOSES: BRIDGING SECONDARY-POSTSECONDARY MATHEMATICS

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Establishing the Content Validity of Tests Designed to Serve Multiple Purposes: Bridging Secondary-Postsecondary Mathematics

In this report, we present an analysis of the content validity of a series of secondary school mathematics tests designed by university mathematicians in California to help improve the articulation between secondary and postsecondary mathematics instruction. Our purpose in conducting this analysis was to improve our understanding of what it means to validate a subject matter test (we shall also refer to these tests as content assessments) that is designed to serve a multiplicity of purposes in the context of the current heightened interest in educational reform. The analysis itself serves as an early prototype in our work on developing models for validating such tests. Thus, while the actual examination was specific to the tests under consideration, both the methods employed in the analysis and the issues raised in conducting them are intended to serve a more general purpose of providing guidance to other educational reform and improvement efforts that place heavy reliance on content assessments and are therefore concerned with the content validity of the tests they use.

In the first tow sections of the report, we comment on the educational context and conditions that impact on the validity and utility of content assessments within the educational improvement movement. Without such background discussion, we believe that it would be hard to understand why a reconsideration of certain test validity notions is warranted. Essentially, we argue that subject matter tests (a) are in the mainstream of reform efforts urged by often competing constituencies and (b) are being asked to serve a multiplicity of sometimes vague purposes, justifing a fresh look at the circumstances that lead one to judge a test as valid for its intended purposes.

In the second section, we define both the terminology and the terrain for our investigation and present our working model for establishing the content validaty of multipurpose subject matter tests. The general framework for the study of which this work is part (namely, the Mathematics Bridging Study) and a brief description of the tests we examined (namely, the Mathematics Diagnostic Testing Project (MDTP) tests) are presented next. In Section 4, the procedures employed in the application of our model to the MDTP tests are described, and the results of our analyses are reported. The paper concludes with the identification of several issues raised by the analyses and brief consideration of their implications for further work on validating content assessments intended to serve multiple purposes.

Background and Context

An expanded role for student testing in the educational improvement process is integral to virtually all the activities generated by the various reform reports of the early 1980's (e.g., California State Board of Education, 1985; National Commission on Excellence in Education, 1983; The National Science Board Commission on Precollegiate Education in Mathematics, Science, and Technology, 1983; United States Department of Education, 1983). Either directly (through the introduction of high school graduation examinations, or upgrading the proficiencies tested in existing competency and proficiency tests, and/or new or modified assessments designed to monitor student and system progress) or indirectly (through their use to hold the educational system accountable for its response to statements of curriculum standards and modifications of the course requirements for high school graduation and college admission), measures of student performance are squarely in the middle of the action.

Unfortunately, the expansion of testing activities has been engendered by often competing political, social, and educational agendas, and thus does not cohere around any clearly articulated focus. If anything, the pressures for educational improvements have served to extend the multitude of purposes that tests have been asked to serve (e.g., Airasian & Madaus, 1983; Anderson, 1986; Burstein, 1983; Burstein & Miller, 1983; Cohen & Haney, 1980; Jaeger & Tittle, 1980; Resnick, 1980) and the diverse forms they take (e.g., Burstein, Baker, Aschbacher, & Keesling, 1985; Bock & Mislevy, 1986; Herman & Dorr-Bremme, 1986). Without a good deal of thought, care, and political savvy, these conditions virtually dictate confusion rather than clarity. Too often, testing efforts introduced under such conditions yield little more than misinterpretation and, even worse, misrepresentation and mischief (Baker & Herman, 1986).[1]

Given the importance of tests in reform efforts and the new emphases on a multiplicity of purposes, it is appropriate to revisit the issue of content validity and how it might apply in this new context. In what follows, we present the results of our initial forays into this issue. The content validity of secondary school mathematics tests developed in the Mathematics Diagnostic Testing Project (MDTP) to improve the articulation between secondary and postsecondary mathematics instruction is examined. While the circumstances surrounding the development of these specific tests are somewhat unique, both the reform efforts that led to their development and the multiple purposes for which they are being used generalize to a broader class of testing activities currently underway or on the horizon.

The Validity of Content Assessments

The kinds of educational tests of interest here are ones developed by an agency or organization *external to the class or school settings in which they are administered* and designed to *measure what students know about a subject matter and could conceivably have learned from their schooling experience.* District, state, national, and international achievement assessments (e.g., commercially published standardized tests; minimum competency, proficiency, and graduation tests, regardless of their source; state assessments; the National Assessment of Educational Progress; the IEA achievement surveys; college admissions and placement testing) all satisfy the above restrictions to a certain degree. To reiterate, the key constraints are *subject-matter focus* and *development external to the setting(s) where instruction takes place*.[2] For the remainder of this report, we refer to such tests generically as *content assessments*.

Linkage to instruction.

Content assessments are being used in the educational reform efforts in a variety of ways (e.g., Anderson, 1985; Elliott & Hall, 1985; Kearney, 1985). One recent emphasis in such assessments is on progress monitoring and accountability through their use as achievement indicators (e.g., Burstein et. al., 1985; Congressional Budget Office, 1986; Fetler, 1985; Raizen & Jones, 1985; Seldon, 1986; Smith, 1985). This emphasis is consistent with both a belief that tests are appropriate vehicles for stimulating curriculum advancement (i.e., "drive the curriculum") and an intent to accomplish just that goal.

The testing model implicit in the above purports to accomplish multiple purposes. But when used as outcome indicators for a relatively diverse set of school programs, content assessments generally can reflect instruction only in broad terms and hence are unlikely to directly impact instruction at the point of delivery, the classroom (Linn, 1983, 1986; Shavelson, Webb, & Burstein, 1986; Tyler & White, 1979).

Perhaps as a consequence of their external source and their intended applicability across a range of programs, most content assessments can be criticized about the extent to which subject matter content is adequately represented and the match between the test and local instruction. That is, because they are developed as generalized accountability measures, such tests cannot precisely measure the enacted curriculum at specific sites, which entail complex interactions among available teacher expertise, student entry skills and knowledge levels, and instructional materials. Yet, since a fundamental reason for conducting most content assessments is to improve educational practice and performance, it is appropriate to examine their validity explicitly in the context of instruction.

Valuable ideas about how best to validate content assessments can be found in research that links educational testing explicitly with instructional experiences. This body of research is a central component of the tapestry of major models of school learning (e.g., Bloom, 1976; Carroll, 1963; Harnischfeger & Wiley) and educational productivity (Barr & Dreeben, 1983; Brown & Saks, 1986; Dreeben & Thomas, 1980). Concepts such as "opportunity-to-learn" as used in the IEA studies (e.g., Husen, 1967; Travers, 1985), "content coverage" (e.g., Floden, Freeman, Porter & Schmidt, 1980; Freeman, Belli, Porter, Floden, Schmidt & Schwille, 1983; Jenkins & Pany, 1976; Schmidt, 1983) and "test overlap" (e.g., Cooley & Leinhardt, 1980; Cooley, 1981; Leinhardt & Seewald, 1981; Leinhardt, 1983) place the link between curriculum as presented to students and the tests used to measure performance at the center of any realistic attempts to understand the consequences of educational policies and practices (Burstein & Miller, 1984; Shavelson et al., 1986).

Determining terminology.

The focus here is on improving the validity of content assessments for educational decision-making related to instruction. Despite the recent proliferation of terminology for instructional aspects of test validity, the classic rubric content is preferable to the currently more popular curricular and *instructional* because of the former's generality in encompassing the diverse reasons for asking and evaluating the perennial question "Is the content of the test appropriate?" And while earlier efforts to examine a test's content validity were primarily judgmental, a broader analysis, both conceptually and empirically, is the current standard (e.g., Cronbach, 1971, 1984; Schmidt, 1983). In fact whether it is reasonable to distinguish content validity as a subform of construct validity would be a more enlightened concern (Cronbach, 1984).

Defining the terrain.

Given that the validity of a test depends upon the purpose for which it is used, it is reasonable to ask just what types of instructional decisions are under consideration. The instructional uses of tests most pertinent to our investigation are for *placement* decisions, *diagnosis*, and *curriculum course revision*.

Student placement in precollegiate education may involve within classroom grouping decisions, where text used or rate of instruction varies, or assignment to curriculum track. At the postsecondary level, placement tests are used to permit enrollment in challenging courses or to identify those in need of additional preparation (Frisbie, 1982).

Diagnostic testing within schools may be designed to determine an individual's difficulty with concepts or to assess the instructional needs of subgroups of students. The test results can help identify instructional weaknesses as well as a particular student`s problems. While the major tools for this sort of testing resides in the textbooks and work materials provided by commercial publishers, content assessments, when properly designed and crafted, can serve this purpose as well. And, as in the case of the specific test we consider later on, teachers will use them diagnostically if appropriate steps are taken to encourage and cultivate such use.

The designation of curriculum and course revision as an instructional use of tests may be surprising given our earlier statements about the indirectness of the impact of tests designed for program monitoring and accountability. However, in formative evaluation (Baker, 1985) and in school-based improvement efforts built around "effective schools" (Edmonds, 1982; Good & Brophy, 1986; Purkey and Smith, 1983) and "clinical teaching" (Hunter and Russell, 1981), content assessment serves just this purpose. The distinctive feature of this type of instructional testing is not the substance of the test, but that group-level (work group, classroom, school), rather than student-level, performance data are usually employed (Burstein, 1984; Burstein & Miller, 1983).

What may not be evident is that the same content assessment system can serve multiple purposes. College placement testing represents an interesting case in this regard. These tests ostensibly operationalize the expectations that colleges and universities have for incoming students. When the test's specifications and test results from their graduates are reported back to feeder high schools, the test can serve also to explicitly communicate expectations that foster curriculum revision at the secondary level.

Replicating the process for prerequisite skills and courses for diagnostic use by secondary school teachers (as is the case in the Mathematics Diagnostic Testing Project, the focus of our content validation effort) produces an extensive system of standards and checkpoints to enhance the possibilities for student success. By following this route, theoretically at least, college placement testing is at the endpoint of a fully articulated system and can provide potentially potent mechanisms for influencing the high school curriculum and improving the articulation of the high school and college curricula. This is the terrain of the current investigation.

Multiple Meanings of Content Validity

What are the problems educators experience in using tests for these types of instructional decisions? What can be done to make them more useful? In our view a significant problem resides in fundamental inadequacies in the content (the "what") of tests intended for instructional decisions. Content assessments need to be designed in such a way that they capture (or encourage) neither a piecemeal accumulation of distinct bits of knowledge or skills (as in some competency oriented testing activities) nor a general, instructionally insensitive capacity (as with most norm-referenced standardized achievement or aptitude tests). Instead, such assessments should portray consequential components of subject matter knowledge that can be linked directly to instructional opportunities and activities (Bock, Mislevy, & Woodson, 1984; Shavelson et al., 1986)

How should a content assessment be validated? The appropriateness of its content for the purpose for which the test would be employed is obviously the primary consideration. But when the same tests can serve multiple instructional purposes within a climate of competing educational, social, and political agendas, a seemingly straightforward activity takes on multiple meanings.

Internal analysis of test content and structure

Presumably, the design of every content assessment begins with a statement of specifications of the content to be included. Historically, these statements have typically started with some form of content and process grid modelled after a taxonomy of educational objectives (Bloom, 1956; Bloom, Hastings, & Madaus, 1971) or more modern variants from a criterion or domain referenced measurement perspective (e.g., Popham, 1981) or, more recently, from a cognitive psychology-based, information processing perspective (e.g., Baker & Herman, 1983; Haertel & Calfee, 1983; Romberg and Carpenter, 1985).

While documentary evidence is hard to find, conventional wisdom is that while most test developers (whether commercial or governmental) begin from reasonably precise specifications of one sort or the other, both item generation difficulties and the empirical activities typically employed in item selection often lead to drift away from certain "cells" of the specifications, especially those that are harder to measure through selected response formats (e.g., problem solving, critical thinking aspects of subject matter). Moreover, specifications are seldom precise when it comes to the distribution of content and processes across the grid nor the appropriate sequencing of content within a set of tests designed to measure across a span of courses or grades.

In our view, matters such as the distribution of content/process questions and its sequencing are as important as the actual elements in the grid. In an earlier investigation of the content of state assessments (Burstein et al., 1985), there was greater between-state variation in the depth and breadth of the content assessed than there was in the topics chosen per se. By depth we mean the inclusion of a sufficient number of test items within the topic (and subtopic) to be able to assert that the content label attached to the topic is a valid descriptor of what might be represented by a student's score on the topic. For example, are there a sufficient number of items to infer whether the student can carry out conversions between decimals, fractions, and percentages?

In our terminology, breadth refers to the adequacy of the representation of the range of conceivably important subtopics within a topic and of the range of topics within the content area for the chosen subject matter/grade/age span. For example, "fractions" at the middle school level includes both common and decimal fractions; thus a test designed for this level should include items representing both subtopics. Likewise, it is inconceivable to exclude entirely ratio, proportion, and percentage problems from tests covering eighth grade mathematics that include fractions topics since the former is directly linked with the latter in typical instruction in the middle school years.

The appropriate stepping/sequencing issue works similarly to the points about depth and breadth. When the content span is broad as in eighth grade mathematics in the United States (McKnight et al.,1987), this concern translates into making sure that, depending on the test's purposes, items should be included which measure what students might minimally know through what a significant segment of students might have been taught. At this age level, the range is especially broad since there are still significant numbers of students doing arithmetic while many at this grade level are taking elementary algebra courses (Crosswhite et. al., 1985). Thus a test that serves a diagnostic function at this level needs to be especially broad.

The issues in examining the internal coherence of a content assessment are complex. Hence, the array of possible strategies for conducting internal analyses of such tests is likely to be large and diverse. Here, again, matters of purpose and the nature of the desired instructional decisions and instructional consequences will determine the chosen strategies in any specific instance.

Actual/implemented curriculum

Three external sources of evidence would seem to be pertinent to ascertaining the content validity of a given educational assessment for a specific set of instructional purposes. First, content assessments can be examined with regard to their match to the actual or implemented curriculum. But, while a close fit to what is actually taught is certainly a desirable property of any educational test, the effort to validate content assessments at this level of specificity is fraught with problems. The nature of evidence required depends on one's choice of definition of actual/implemented curriculum and on one's willingness to employ potentially costly means for its measurement (e.g., classroom observations, teacher logs, teacher surveys, textbook analyses) Morever, each of the alternative measurement methods mentioned pose different tradeoffs among psychometric quality, administrative burden, demands on teacher time and costs. Under such conditions, it is little wonder that the matching of tests to actual/implemented curriculum has remained primarily a research rather than an operational exercise.

Curriculum expectations: Explicit desired/intended curriculum

A second source of external evidence comes about through efforts to match content assessments to the desired, or intended, curriculum. Here the question is:

> Does the test measure the significant knowledge and skills enunciated in curricular standards, i.e., what people say students should learn or know?

Desired/intended curriculum is derived from the curriculum expectations expressed in curriculum guidelines (continua, syllabi, scope-and-sequence statements) and in various forms of public statements about curriculum standards (e.g., the Model Curriculum Standards adopted by the California State Board of Education (1985)) and academic preparation prerequisite to college (e.g., the academic competencies articulated by the College Board's Project EQuality (1985), Agenda for Action from the National Council for Teachers of Mathematics (1983) and Educating Americans for the 21st Century produced by The National

Science Board Commission on Precollegiate Education in Mathematics,Science, and Technology, 1983).

As the numerous cited examples may suggest, however, a critical question is _whose expectations_ from among a myriad of choices should the test match? For example, in the case of content assessments designed to serve as a bridge between secondary and postsecondary mathematics, precollegiate and postsecondary educational institutions and governing bodies, different governmental levels, professional organizations, testing organizations, mathematicians, mathematics educators, and curriculum specialists are all sources for these expectations.

Other Tests: Implicit Desired/Intended Curriculum

Other external indicators of performance in the targeted subject matter are the third possible source of external evidence of content validity. Here, the question is:

> Does the test measure the significant knowledge and skills embodied in other accepted indicators of quality in the subject matter area?

Once, again, the range of possibilities is considerable, at least in the abstract. The contents of commercially published standardized tests, criterion referenced testing item banks, state assessments, minimum competency tests, the National Assessment of Educational Progress, college entrance examinations, and tests from the international surveys of educational achievement (IEA) could all be conceivably compared with those of a given content assessment.

Operational and Practical Considerations

In practice, the scope of the content validity evidence one gathers should be balanced by instructional consequences of the content assessment. The necessary precision for the instructional decisions to be made should be the basis for judging the adequacy of the validation effort. In cases where decisions based on test performance are exclusionary, as in placement into curriculum tracks and courses of study, the evidentiary demands should be high. When other sources of information are also involved in arriving at instructional decisions (e.g., the teacher's own testing and observations with respect to diagnostic decisions), the acceptable margin of error is likely to be larger. Thus the investment in evidence gathering should be relatively modest, assuming that the teacher is capable of accurately judging student competence and skill.

Within the constraints of the kinds of content assessments under consideration here, and their possible instructional uses, we believe that the content validation effort should focus primarily on examining the validity of their internal task analysis and test structuring schemes and on comparisons with curricular expectations and other external indicators. These sources of evidence are most consonant with the intent of the specific content assessment to be examined below and best balance the tradeoffs in human and economic costs to achieve the desired goal.

Mathematics Bridging Tests

The content validity analyses to be discussed below were conducted as part of the Bridging Secondary-Postsecondary Mathematics Study, or more succinctly, the Mathematics Bridging Study. Before proceeding with the discussion, we briefly describe the main study and the origin and characteristics of the specific content assessment whose validity evidence we examined.

The Mathematics Bridging Study

The Mathematics Bridging Study is designed to investigate the feasibility of developing a mathematics placement-diagnostic testing system that is articulated with both the desired and implemented curriculum at the secondary level and with necessary competencies for successful performance at the postsecondary level. To improve the articulation of mathematics instruction between secondary and postsecondary institutions, bridging tests should be designed to measure the content of secondary instruction in mathematics while at the same time, measuring the prerequisite competencies and skills necessary for pre-calculus and calculus coursework at the university. Moveover, such tests must be divisible into instructionally meaningful components or skills that can be used as separable indicators in the placement, diagnostic, and curriculum revision process. If these goals were realized, the resulting test could serve as the needed content-sensitive bridge between mathematics instruction at the secondary and postsecondary levels.

The Mathematics Diagnostic Testing Program

Our efforts to investigate the properties of a multipurpose bridging testing system led us to form a working arrangement with the Mathematics Diagnostic Testing Project (MDTP). MDTP was launched by university mathematicians in California to tackle problems in articulation between secondary and postsecondary institutions. They did so in response to the recently highlighted situation wherein many students enter college poorly prepared to take college level mathematics or other courses with prerequisites in mathematics.

MDTP was originally designed to be a collaborative effort by the two public university systems to develop placement examinations in response to heavy demands for remedial mathematics. In addition to their placement function, the MDTP leadership also viewed these tests as a means to document student preparation in mathematics and to inform improvement efforts at the secondary level by reporting performance results to feeder schools. This effort eventually evolved into its current form wherein diagnostic information about student preparation in secondary school mathematics classes is obtained from tests developed and scored by MDTP. This testing service is provided at no direct cost to secondary school teachers and students who voluntarily participate.

Currently, the MDTP is a unique undertaking.[3] It is designed to span the chasm that normally exists between mathematics preparation deemed essential for collegiate study and that obtainable from current secondary school course offerings. This "bridge" is designed to serve at the same time as a tool for increasing articulation and a means for valid instructional diagnosis and for better placement.

One of the most unique aspects of MDTP as currently operated lies in its direct link with secondary school teachers, almost totally bypassing the educational bureaucracies at the precollegiate level. The program is funded as a separate line item of the university-based California Academic Partnership

Program. MDTP currently administers four one-hour tests (all multiple choice) covering Algebra Readiness, Elementary Algebra, Intermediate Algebra and Precalculus. Teacher participation is voluntary. When teachers choose to use the MDTP scoring services, the profile of results (for approximately 5 topics as well as item-level data for the whole class) is reported within a week's time directly to the teacher for his/her own use. Results are never reported for levels above the classroom; teachers control access to their classes' results other than for the research and evaluation purposes of MDTP. This combination of teacher empowerment and bond-building between university mathematicians and secondary school mathematics teachers has resulted in exceptional levels of voluntary demand for the services, repeat business, and expressions of teacher satisfaction. Roughly 600 teachers and 150,000 students participated in MDTP testing during the 1985-1986 academic year.

While concrete, explicit statements about what MDTP intends to accomplish are hard to locate, it is not hard to envision what the university mathematicians hope will be the consequences of their efforts as well as those of other mathematics reforms in California. Namely,

(1) Teachers in lower-level mathematics classes will undertake the necessary remediation to ensure that their students have a firm grasp of the mathematics necessary to succeed in higher level mathematics;

(2) Students going on to higher level mathematics courses enter these courses better prepared;

(3) Teachers in higher-level mathematics classes adjust their teaching to accomodate improvements in students' mathematics preparation; and

(4) students exiting from the secondary school mathematics are better prepared for college.

Given these desired goals, an examination of the relationship of the contents of the MDTP tests to other curriculum enhancement and testing activities serves to illustrate a content validity model for assessing multipurpose bridging tests.

Content Validating the MDTP

Below we illustrate a model for the content validation of bridging tests based on internal analyses of the MDTP tests and a comparison of their contents with a variety of statements of desired curriculum and preparation at the secondary and postsecondary levels and with other mathematics tests given at the secondary level. Tests developed by the MDTP work group will be examined for their coverage of the intended and implemented curriculum of secondary school instruction and their relevance to the content of university courses.

The content validity analysis of the MDTP test items for measuring what should be taught at secondary school mathematics was done by analyzing whether the tests have sufficient breadth and depth of coverage to measure the desired curriculum and how well they compared with other tests spanning the same domain. The desired curriculum was determined by examining various recommendations for secondary school mathematics and for preparation for college work. Four different sources of curriculum expectations were considered:

a. National Science Board Commission on Precollege Education in Mathematics, Science, and Technology (1983), Educating Americans for the 21st Century. Washington: National Science Foundation.

b. College Entrance Exam Board (1985). Academic Preparations in Mathematics: Teaching for Transition from High School to College. N.Y.: College Board.

c. California State Board of Education (1985). Model Curriculum Standards: Grades 9 through 12. 1st Edition: Sacramento: California State Department of Education.

d. California State Department of Education (1985). Mathematics Framework: for California Public Schools Kindergarten Through Grade Twelve. Sacramento: CA State Dept of Education.

The contents of the MDTP tests were also compared with three other external indicators. The most extensive comparison was made with the content specifications and test items from the Second International Mathematics Study (SIMS). SIMS tests were administered in 1981-1982 to random samples of approximately 250 8th grade classes and 250 12th grade classes taking mathematics at at least the level of intermediate algebra. Second, the MDTP items were compared with the cells of the 8th grade mathematics grid contained in the Bock and Mislevy (1986) paper on the duplex design for comprehensive educational assessment for the states. This grid was developed in a cooperative effort with state mathematics curriculum and testing specialists from California and Illinois. Finally, the MDTP content classification was compared with that from the California Assessment Program at Grades 8 and 12 (California Assessment Program, 1984, 1986).

Procedures

As delineated above, three types of evidence contributed to the content validity analyses of the MDTP tests:

MDTP TESTS INTERNALLY

MDTP vs CURRICULUM STATEMENTS

MDTP vs OTHER TESTS

Below we briefly describe the procedures employed to carry out each type of analysis.

Internal Analysis of MDTP Tests. As mentioned earlier, currently there are four levels of MDTP tests (Algebra Readiness, Elementary Algebra, Intermediate Algebra, and PreCalculus). At each level except Algebra Readiness (which was used officially for the first time this year), three essentially parallel forms (labeled A through C at each level) are administered. In fact, roughly 80 percent of the items appear on all three forms at a given level.

Test items were written by members of the MDTP working group according to a general set of test specifications devised by the group. From a psychometric, test development perspective, the level of specificity of the item writing guidelines appear uneven, especially when viewed across forms. This unevenness is apparently due to the working group members' strong committment to both measuring concepts and skills viewed as important for later mathematics training and to using only strong items where the concepts being measured could be clearly identified. As a consequence of the large content span of the four test levels, one also finds rather detailed specifications of certain topics on the lower level tests (e.g., arithmetic algorithms and operations) which are only lightly covered in upper level tests. The opposite phenomenon is also in evidence with respect to the topic algebraic equations, for example.

Another obvious feature of the tests and test specifications provided by MDTP is that the working group members did not formally use the classic cross-classification of content and process dimensions. Instead, the tests were built almost entirely around concepts and algorithmic applications. The primary means for introducing higher cognitive level test items were through brief word problems and graphical representations of algebraic algorithms. Very few multi-step problems were employed. We will return to these observations later in our analysis.

In our work, we began with test specifications provided by the MDTP working group. We decided early on that the set of topic categories to be used for item classification needed to span all four test levels. Moreover, for a variety of reasons, we wished to use as few categories as possible while still conveying the nature of the mathematics tested. Since roughly

150 distinct topics were contained within the test specifications across the four levels (listed as separate categories at at least one level) and there were only about 220 unique items across all test forms, it took several iterations to achieve staff consensus on a set of topic labels at our most precise level of specificity. Furthermore, as part of the process, we decided to abandon attempts to directly represent the process/cognitive level dimension in our topic list but rather to simply use some means to designate where word problems and graphical representations of algebraic algorithms occurred.

The end result of this effort was a set of 13 topics at the highest level which included 21 subtopics which in turn subsumed 84 subtopics at the most detailed level of specificity. The resulting scheme for classifying MDTP test items into our version of the MDTP grid is depicted in Tables 1 through 3 which will be discussed below. This grid also was put to use in later examinations of the curriculum statements and other tests, activities that will be described in the following sections.

MDTP vs. Curriculum Statements. As mentioned earlier, the contents of the MDTP tests were compared with four separate statements of curriculum expectations. The procedure went as follows:

1. Starting from the MDPT classification scheme, each element of the four curriculum expectation statements (reproduced in Appendies A through D) was assigned to one or more entries in the scheme if possible. Otherwise, the expectation statement was assigned to a list of topics not included on the MDPT scheme.

Each element of each statement was given an ID letter and/or number, which was then recorded in the appropriate box indicating a match between an MDPT topic and one of the curriculum statements. Some statements contained more than one element that matched a given MDPT topic. Likewise, some curriculum elements were actually rather broad, and thus might be matched to more than one MDPT topic. Finally, some curriculum statements were so vague as to make matching somewhat tentative, such as the element under algebra, development of problem solving abilities. Presumably this refers to the ability to do word problems using the types of algebraic skills mentioned in prior elements in this statement; however, the domain has not been clearly specified.

2. After attempting to match all four curriculum statements to the MDPT scheme, all elements of each statement were reviewed and checked. (Note: Some curriculum statements were extremely vague, and experience with the language of other statements facilitated interpretation in some cases.)

The procedure outlined resulted in two products:
(1) a table of elements in each of four Curricular Expectation Statements by MDPT topics (See Table 4 below).
(2) a list of additional elements of curricular expectation statements that do not match any of the MDPT topics (Appendix B through D).

MDTP vs. Other Tests. There were three sources of other tests or test specifications used in our content validity analysis. First, the tests used in the Second International Mathematics Study (SIMS) and their content-process grids were examined. While these tests were administered to 8th grade and 12th grade samples in the United States, their design was such that they basically span the same range of content as the MDTP tests. The main differences between them are that the SIMS 8th grade test contains more items devoted to whole number arithmetic and the 12th grade SIMS test covers topics that are typically introduced in Calculus courses. Our strategy with SIMS was to classify both SIMS' own items and MDTP items into the cells of the SIMS grid. In this way we expected to obtain some indication of degree of correspondence. Both simple counts of items per cell and the detailed arraying of specific MDTP items to cells in the SIMS grid were carried out and are reported in Tables 5 through 8 below.

The second source of other test specifications was actually simply a content grid for eighth grade mathematics developed by Darrell Bock and his associates in collaboration with state curriculum and testing personnel from California and Illinois. Tables 10 and 11 report these results. In the case of the classification of items from both tests, cell entries will be simple counts of items from a given test source or form.

The third source of test-generated comparison for the MDTP tests are the content specifications for the mathematics tests administered by the California Assessment Program (CAP). CAP tests have been administered by the California State Department of Education for a number of years; at the secondary level, all 8th and 12th grade students are assessed. The 8th grade content specifications were established in 1984; there was some linkage between the CAP test at this level and the SIMS tests. CAP is currently revising its 12th grade mathematics test so it may not be possible to use it for our comparisons at this time.

In this comparison it was not possible to examine individual CAP items. Given these circumstances, we were only able to report on the distribution of CAP items among its topics as reported in the most recent CAP annual report (1986). Thus the comparison in this case is crude at best.

Insert Table 1 here

Results

The tables presented in this section are organized around the process of establishing content validity proposed here: internal analysis, comparison with curriculum statements, and comparison with other tests. The discussion of specific results will be brief since this exercise is intended merely to illustrate our approach to content validating a multipurpose test.

Internal Analysis of MDTP Tests. Tables 1 through 3, which follow, are concerned with which topics are covered by the four levels of MDTP tests and how the test items are distributed across topics and levels. Specifically, the tables address the following questions:

Table 1	Which topics are covered by each of the four MDTP tests?
Table 2	For each MDTP test, how many items cover each of the major topics?
	For each test, how many different subtopics are covered by those items?

Table 3 Of the 21 different topics covered by the four MDTP tests, ...

How many topics were covered on each test?

How many topics were covered by each pair of tests?

How many topics were unique to each pair of tests?

Table 1- 3 here

Our interpretation of the results from this set of tables draws intermittently from all three rather than from each one separately. In general, there were several distinctive patterns. Test content tended to progress sequentially and logically across the four levels with both the Algebra Readiness and Elementary Algebra tests covering topics normally considered to be high-level arithmetic (e.g., fractions, conversions among fractions, decimals and percent; exponents; arithmetic operations with simple polynomials) and beginning algebra and equivalent geometry and measurement topics (e.g., linear equations, measurement formulas, properties of triangles). About half the topics were covered on several tests and half were covered only on one test. The level of specificity of coverage within these topics is more detailed than on the two upper-level exams. In fact, the Intermediate Algebra and PreCalculus tests tend to span a lot of material but in a lightly sampled manner. For example, there were still a few items from the exponentiation and polynomial topics on the PreCalculus test along with heavy coverage of relatively advanced functions topics.

The degree of sequencing and overlap in topics is best illustrated in Tables 2 and 3. The distribution of items to topic evolves as one moves across test levels and up the rough content hierarchy exhibited in Tables 1 and 2. There is a steady decay in the amount of topic overlap as the separation between test levels increases (Table 3). A few topics are unique to a pair of levels.

Looking over the test items from a process perspective, other patterns tend to emerge. There are lots of algorithms represented at all four test levels with little emphasis on estimation and approximation skills. Computation, in and of itself, tends to be minimized; rather recognition and application of algorithms is the key underlying skill that these tests attempt to assess. So-called critical thinking/problem solving skills are not particularly well represented except through relatively short, straightforward word problems that are sprinkled throughout, especially at the lower test levels. There is, however, considerable attention to graphical representations of various algorithms. This is somewhat surprising given the obvious attempts to cover a broad array of content in an efficient and parsimonious way, as reflected in the 40-50 item tests designed to be taken during a single class period.

In general, then, the internal analysis suggests that the MDTP's working group's interest in efficient assessment of the knowledge and skills that students are expected to learn prior to college-level mathematics is faithfully transmitted through their tests. No topic receives too substantial coverage; one can not hope to be deep when the target body of content is so broad and testing time so limited. Not much time is devoted to complicated multistep problems and long word problems; the former might be awkward to fit within the diagnostic intent of the tests and the latter simply take too much time. What little emphasis enters in on alternative representations of concepts and algorithms occurs largely through graphical representations of algebraic algorithms. This is a sensible choice of what some might view as abstraction (or concreteness, depending on one's perspective). As students progress in mathematics, the ability to operate with both algebraic and graphical representations of problems becomes highly prized.

MDTP vs. Curriculum Statements. The MDTP test topics are compared with the four different curriculum expectation statements in Table 4. These statements have been appended to this document (Appendices A-D). The results in this table address the following questions:

> Which MDTP topics match which elements of the curriculum statements?
>
> Which MDTP topics were NOT addressed by the curriculum statements?
>
> On which MDTP topics was there agreement among curriculum statements?

Table 4 here

Examination of Table 4 reveals that some curriculum elements cover several diffferent MDTP topics and are thus entered in several cells of the table. Some of the elements of the NSF and the College Board statements used identical wording and thus matched the same MDTP topics. As might be expected, the Model Curriculum and Framework statements were quite similar to each other, although the Framework included information about the Junior high school level, some of which was not covered by the Model Curriculum statements (only grades 9-12).

We encountered substantial difficulties in matching some curriculum statements to the MDTP grid. The NSF curriculum statements were the most troublesome to match because of much vague and broad wording. In the appendices, we have tried to convey the difficulties in matching. A '+' before a statement indicates that part of an element matches the MDPT topic but one or more significant parts of the element are not contained in MDPT topic. For example, from the California Mathematics Framework, statement J-N8 calls for finding the square roots of whole numbers, fractions and decimals whose square roots are rational; and to estimate irrational square roots as a check of results obtained with calculators. This element overlaps the MDPT topic "square roots of perfect squares" but entails much more than is not covered by MDPT topics.

There was also a considerable amount of non-coverage of elements from certain curriculum statements by the MDTP grid. The non-covered elements are marked with an '*' in the appendices. Fifteen of the 41 elements from the NSF statement, 8 of the 24 from the College Board, and 19 and 32, respectively, from the Model Curriculum Standards and Mathematics Framework are not clearly represented in the MDTP grid. Some of the non-coverage is understandable since, for example, the Framework includes topics in logic and probability and statistics, which the MDTP tests do not assess. The other aspects of non-coverage are the result of the tendency of specific statements to characterize processes and procedures (e.g., "using calculators to ...") rather than content per se.

MDTP vs. Other tests and Categorizing Systems. The final group of tables, Tables 5 through 13, summarize comparisons of the MDTP test items with several other tests and item categorizing systems:

- MDTP vs. SIMS
- MDTP vs. Bock's Duplex Grid
- SIMS vs. Bock's Duplex Grid
- CAP

The discussion of results will be divided along these same lines.

1. MDTP vs. SIMS. Tables 5 - 9 analyze the content of the SIMS items and compare the MDTP with the SIMS categories. The questions addressed by these tables include:

Table 5	How many SIMS items match each SIMS category (topic) across three hierarchical levels of cognition for the 8th grade population? I.e., what skills and topics are tapped by the SIMS(8)?
Table 6	How many SIMS items match each SIMS category across three cognitive levels for 12th grade? I.e., same as Table 5 but for 12th grade.
Table 7	How many items from each MDTP test match each of the SIMS categories, at each cognitive level?
Table 8	Focusing on the Algebra Readiness test of the MDTP battery, which items cover each category, at each cognitive level?
Table 9	Focussing on the Elementary test from the MDTP battery, which items cover each category, at each cognitive level?

Tables 5-9 here

Contrasting first Tables 5 and 6 with Table 7, there is considerable overlap in topic coverage in most areas, especially in the Arithmetic and Algebra sections of the SIMS grid at the 8th grade level. In fact the lower three levels of the MDTP battery are well-reflected in the 8th grade SIMS grid, perhaps a reflection of the pace of American secondary school mathematics compared with other countries. There is also especially spotty representation of Geometry and Measurement topics. The former might be explained by the fact that the SIMS grid in this area spans a broad range of topics, few of which are universally part of the geometry curriculum across countries (see McKnight et. al., 1987). The measurement results reflect the distinctions in the emphasis on metric topics plus the fact that much of the MDTP topic coverage at the lower levels is targeted to preparation for and in algebra. Tables 8 and 9 reinforce these impressions about the lower levels of the MDTP battery.

The match at the SIMS 12th grade level is much less pronounced; essentially the match occurs only in Algebra topics, trigonometry, and elementary functions in the Analysis area. This result was not unexpected given the considerable emphasis on Calculus topics at 12th grade SIMS, which remains largely uncovered by the MDTP test battery.

2. MDTP and SIMS vs. Bock's Duplex Grid. Table 10 compares MDTP items to the DUPLEX categorizing system, which is based on two stated assessment plans for which items are not yet available for direct comparison with the MDTP. This table addresses the question:

> How many items from each MDTP test match each of the DUPLEX categories, at each cognitive level?

Table 10 here

Table 11 compares the SIMS items and categorizing system to the DUPLEX system. It addresses these questions:

Which SIMS categories match the DUPLEX categories?

How many SIMS items match each of the DUPLEX categories?

Table 11 here

Here, again, the MDTP tests are spotty in Geometry and Measurement and devote no items to probability and statistics at all. But the Algebra topics are well represented across all four MDTP tests, albeit at systematically varying levels of difficulty. In contrast, the longer SIMS test item set covers virtually every topic in the grid at all three cognitive levels.

One notable point that becomes more evident from classifying the MDTP items into the Duplex grid is that for the most part, the MDTP tests measure procedural skills (algorithmic applications primarily) and what Bock and Mislevy call higher level thinking. The latter reflects primarily graphical representations and short word problems as identified earlier in the internal analysis. Across all four tests, we found very few items that could be clearly classified in the factual knowledge category (e.g., terms, definitions, concepts); those that were were heavily weighted toward fractions and geometry topics.

One other point about the Duplex grid is worth mentioning. We experienced some difficulty in handling MDTP topics such as exponents and powers and absolute values. Different mathematicians and mathematics educators define topic boundaries differently. Sometimes a topic label is too telegraphic to enable one to detect whether the term is being used consistently. We believe that that is the case here. While they elaborated on what the cognitive level category labels meant in their paper, Bock and Mislevy (1986) apparently assumed that the content categories were self-explanatory. In general (and in this specific instance), this assumption is likely to be unwarranted.

3. CAP. Tables 12 and 13 present the content of the CAP math items according to the CAP categorizing system. Analysis of and comparisons to this test are limited by the unavailability of the items for scrutiny. We do, however, provide the more detailed content specifications for the 8th grade CAP in Appendix E. These tables address the questions:

> How many (% of total) CAP items cover each of the CAP categories (Grade 8)?
>
> How many CAP items cover each of the CAP categories (Grade 12)?

Table 12-13 here

The match of MDTP with CAP is difficult. CAP specifications, especially the detailed ones in the appendix, tend to broadly inclusive and written to cover skills that are evidenced across several subtopics. Moreover, there are specifications in the area of problem solving, clearly not reflected in the MDTP grid or in many of its items.

These difficulties notwithstanding, our sense is that CAP and MDTP items match well in upper-level Arithmetic topics (CAP's categories of Numbers and Operations vs. MDTP's Fractions, Decimals, Proportion, Percent) and in lower level Algebra topics.

The match is poorer in Geometry and Measurement for reasons already stated, and non-existent in CAP's categories on Probability and Statistics, Tables, Graphs, and Integrated Applications, and Problem Solving.

The above discussion focuses on 8th grade CAP. At the 12 grade CAP, the situation is much worse unless one compares with the MDTP Algebra Readiness tests only. But this is precisely the most reasonable comparison. The version of the 12th grade CAP included here is essentially a basic skills test, designed to measure the mathematics all students might be expected to know upon high school graduation rather than the contents of high school mathematics courses taken in preparation for college. Once the CAP 12th grade test is revised to better represent the whole of secondary school mathematics (this revision is currently underway), one could expect the match to be more comprehensive.

Issues Raised by the Content Validity Analysis

Without going into too great a detail regarding the specifics of our content validity analysis of the MDTP tests, we did encounter several issues that suggest that the task of content validating a multiple purpose test of this sort is not likely to be a straightforward task. Some of the reasons for the difficulties include the following:

a. In the first place, we decided to define the content spanned by all four MDTP tests since our goal was to examine the validity of the use of these tests in preparing secondary school students for postsecondary mathematics. With this goal in mind, it is not sufficient to attach one of the MDTP tests purely to this goal and specifically to any single secondary school mathematics course. Students following different routes through the secondary school mathematics curriculum can still be judged from the perspective of the expectations for pre-collegiate mathematics preparation even though those choosing different college majors might require quite different levels of knowledge and might receive substantially different postsecondary training. The knowledge level of interest here might be termed "high minimal standards" in mathematics for the college-bound student rather than for either the mathematical sciences specialist or the general populace of secondary schools.

b. The decision to develop a content classification that spanned the four tests introduced certain additional complexities. While there were clearly overlaps in the contents of adjacent tests at a general topic level, the more fine-grained the specification, the smaller the overlap became. In part this arose because skills from the lower level tests appeared in less precise specification in higher level tests; this was only natural as the higher-level tests typically included items that involved the combination of multiple lower-level concepts and skills. Moreover, an "increasing degree of difficulty" factor also was built in to reflect the assumed increased sophistication of students who would be taking the upper-level tests.

c. After several iterations, we managed to "narrow" the content classification scheme to approximately 85 separate elements and classify the approximately 220 items from the four MDTP tests, accordingly. Our separate elements were primarily content focussed (i.e., specific concepts, skills, and algorithms). Graphical/figural/geometrical representations of algebraic and algorithmic concepts juxtaposed with their symbolic and numerical counterparts and simple word problems were essentially the only attempts to move into the critical thinking, problem solving domain. Our sense was that this is about as close as the university mathematicians who develop the MDTP tests ever come to a process-oriented specification of their test contents.

d. The effort to compare contents of two tests designed for different purposes and by different groups is hard enough in and of itself (we found this out in attempts to compare state assessment tests). We stretched this effort much further by attempting to compare not only tests but tests with distinct curriculum expectations expressed at various levels of specificity and generality. Different groups also speak about about the same content using quite different language. There simply is no single universal classification of the "possible" content of secondary school mathematics. Thus we have no universally agreed upon standard to which to compare our own effort. While our work is far from "seat of the pants", it is not by any stretch of the imagination, straightforward.

e. California's leadership in tackling the improvement of the secondary school mathematics instruction has created its own special problems for our validation efforts. On the one hand, the whole MDTP effort is forward thinking in that the bridge to better articulation has been clearly established by the postsecondary sector; there is a real committment by university mathematicians to improving secondary school mathematics instruction that is still exceedingly rare in other content areas and in other parts of the country. The MDTP tests, the focal point of this bridge, truly reflect the knowledge and skills which the persons responsible for mathematics course offerings in two four-year public university systems believe are needed for postsecondary mathematics preparation. The tests and the content specification behind them were prepared from the perspective of the postsecondary discipline as they believed they should be conveyed to secondary school teachers.

On the other hand, reform efforts evolving through mechanisms established by the California State Board of Education and the California State Department of Education are driven by the field as defined by mathematics educators and the mathematics associations for secondary school mathematics teachers and curriculum specialists. The directions these groups are aiming are less strictly focussed on traditional content issues. Rather their statements evidence more emphasis on the processes for learning mathematics and for enabling students "to understand mathematics" and become better problem solvers. Less emphasis on computational and algorithmic skills (because calculators and computers will do the actual number crunching) and more emphasis on "number sense" (e.g., mental mathematics such as estimation and approximation skills) are today's slogans for this segment of the mathematics community. They also champion building into the curriculum data analytic skills fostered by more probability and statistics training. The recommendations of this precollegiate sector of mathematics educators has already begun to creep into the state testing activities in California and with the new textbook adoptions will become a more central part of the curriculum.

The efforts described thus far are operating on separate tracks with the same general goals and purposes (improved mathematical understanding and preparation) but with different presciptions for achieving them. At the secondary level, mathematics teachers, and as a consequence, their students, are caught in the middle as it is unclear whether the knowledge/skills/product focus of the postsecondary mathematics sector is consistent or in conflict with the understanding/number sense/process thrust of the precollegiate mathematics education leadership.

Realistically, we are unlikely to be able to deal with the latter question until additional data are collected in sites where the precollegiate defined reforms are clearly in place but MDTP tests are still administered. In the meantime, however, the apparent conflict raises questions about how much we should expect overlap in the curriculum expectations between MDTP tests and other reform efforts.

At this point, our belief, developed in part from the MDTP content validity analysis, is that if test information is referenced to the subject matter domain from a disciplinary point of view and to the actual instruction students receive, then the potential for improvement in student performance is more probable. Focus on specific modifications in instruction is more likely when a measure is specific rather than when the measure reports general student capacity.

Concluding Comments

Reflecting on the process by which we arrived at Tables 1 through 3, several obvious points about the difficulties of developing consistent guidelines for internal analyses of content validity were revealed. As became even more evident in our later comparisons with other tests, the task of defining "a topic" is hardly clearcut. Our tables reflect topics at varying levels of specificity and even our attempts to aggregate content into larger bundles in certain analyses were not completely satisfactory.

Topic labeling is another fuzzy activity, a problem again accentuated throughout our analyses. We went through several iterations of the content classification scheme to remove as much ambiguity as we could. While the current labels were generally sufficient to tell what type of questions might be included within their "cell", we are not at all confident that others would label the same topics in the same way (Bock obviously didn't). Nor would different judges classify the individual items in exactly the same way given our chosen topic labels. To the very end, we were still shifting certain items within the grid.

These types of problems would seem to be endemic to analyses of the validity of content assessments. There are certainly subtle nuances in experts' classification and labeling schemes that cannot be captured by any grand, universal system. The categorizer's role, perspective, and personal systems of relative valuing of content invariably intrude as the distinctive, often unique features of specific textbooks and standardized tests built around an essentially common core of content readily attests. Nevertheless, there are certain regularities across the array of classification schemes, tests, and textbooks we have seen, at least in mathematics. Thus attempts at such analyses are hardly futile. Rather the issues are complex but attempts to address them thoughtfully have potential direct and secondary benefits to improving the capacity of content assessment to serve a vital, multipurpose role in educational reform and imporvement.

Footnotes

[1] One cannot help but wonder whether this is the fate in store for the current effort to expand the role of the National Assessment of Educational Progress to serve what may be conflicting political and educational purposes (Burstein, 1986).

[2] Although curriculum embedded tests (in textbooks and workbooks) might be judged to fall within the bounds of the above characterization, we explicitly exclude them from the present analysis. Although such tests are developed externally, their intent is to reflect instruction almost tautalogically under "ideal" circumstances. These ideal circumstances exist only when the teacher chooses to teach a unit from a given textbook and then proceeds to test whether students have grasped the material and the contents of the curriculum-embedded test validly relate to the curriculum material the test is supposed to represent. Recent work by Linn and researchers at the Center for the Study of Reading addresses the current status of curriculum-embedded tests.

[3] There are other ongoing efforts designed to improve precollegiate mathematics preparation and school-university articulation launched by the higher education community. Examples include the development by Ohio State University (Leitzel, 1985) of the Early Mathematics Placement Testing program for high school juniors and new senior-level courses for underprepared students who intend to go to college, and the New Jersey College Basic Skills Placement Test, developed by the state's Department

of Higher Education and administered to every entering public college student (Morante, Faskow, & Menditto, 1985). But we are not aware of any other mathematics preparation reform built around testing that places so heavy a reliance on the willingness of individual secondary school teachers to voluntarily participate.

TABLE 1
TOPICS COVERED AND THE DISTRIBUTION OF TEST ITEMS FROM THE MATHEMATICS DIAGNOSTIC TESTING PROJECT*

	AR	EA	IA	PC	Total
I. INTEGERS					
. Basic operations with signed numbers	7 21 27 5w 8w 22w				6
. Prime factorization	12 20				2
. Number line	9 35	43			3
Block Total	10	1			11
II. FRACTIONS, DECIMALS, PERCENT, RATIO, PROPORTION					
. Equivalent fractions	24				1
. Addition and subtraction of fractions	18 42 43	15			4
. Multiplication and division of fractions	29 11 2 16w 19w	21w			6
. Order and comparison of fractions	37 41				2

NOTES:

1. The numbers in the table refer to item numbers from the alternate test forms used by MDTP. While certain of the tests had three alternate forms, in actuality, a majority of the items were exactly the same across forms and even when the item changed, it typically covered the same topic. When an item on a topic appeared on only a single form, its form number is given following the item number (e.g., "12a" in the Elementary Algebra column that indicates that item 12 on Form A of the Elementary Algebra test measured the topic "multiplication of a monomial with a polynomial".)

2. The test designations are as follows:

'AR': Algebra Readiness
'EA': Elementary Algebra
'IA': Intermediate Algebra
'PC': Precalculus

3. 'r' means the test item is listed under more than one topic.

'w' means the test item is a word-problem item.

	AR	EA	IA	PC	Total
. Simplification of complex fractions	31				1
Sub-Block Total	(12)	(2)			(14)
. Addition and subtraction of decimals	1 3	5			3
. Multiplication and division of decimals	17 25	2b			3
. Estimation and approximation with decimals	32				1
Sub-Block Total	(5)	(2)			(7)
. Conversion between fractions and decimals		2ac			2
. Conversion between decimals and percent	26				1
. Computations with both decimals and fractions, rounding off	45				1
Sub-Block Total	(2)	(2)			(4)
. Percentage	10 44w	29	30w		4
. Proportion	30w	45w	32cw 38w	28w	5
Sub-Block Total	(3)	(2)	(3)	(1)	(9)
Block Total	22	8	3	1	34

III. EXPONENTS, RADICALS AND SQUARE ROOTS

	AR	EA	IA	PC	Total
. Laws of exponents			18 28	10	4
. Powers of 10 and scientific notation	40 47	38	4		4
. Exponentiation with integral exponents	25r 34	4 8	13		5
Sub-Block Total	(4)	(3)	(4)	(1)	(12)
. Square roots of perfect squares	23 48	9			3
. Simplification of square roots		28			1
. Addition and subtraction of square roots		11			1
. Multiplication and division of square roots		24	16r		2
Sub-Block Total	(2)	(4)	(1)		(7)
. Conversion between radicals and exponents				34	1
. Rationalization of the denominator			31c	7	2

	AR	EA	IA	PC	Total
. Addition and subtraction of radical expressions				6	1
. Numerical calculations with rational exponents and radicals			7 16r 31a		2 1
. Algebraic calculations with rational exponents and radicals			31b 23		1 1
. Factoring and simplifying an algebraic expression involving rational and literal exponents and radicals			33	18	2
. Estimation and approximation with radicals		3			1
Sub-Block Total		(1)	(7)	(4)	(12)
Block Total	10	8	12	5	35

IV. POLYNOMIALS

	AR	EA	IA	PC	Total
. Arithmetic operations involving literal symbols	33				1
. Simplification of a polynomial by grouping (1 and 2 variables)		1 22	10		3
. Addition and subtraction of polynomials		22r			1
. Evaluation of a polynomial (1 and 2 variables)	15 4	14	1		4
. Multiplication of a monomial with a polynomial		12a			1
. Multiplication of two binomials		12bc			2
. Division of polynomials				14	1
. Squaring a binomial (a+b)2, (a-b)2 and (a+b)(a-b)		6 23b	36		3
Sub-Block Total	(3)	(8)	(3)	(1)	(15)
. Factoring polynomials by finding a common monomial factor		23c			1
. Factoring a trinomial		13	44		2
. Factoring perfect squares and differences of squares		23a			1

	AR	EA	IA	PC	Total
. Complex numbers			42		1
Sub-Block Total		(3)	(2)		(4)
Block Total	3	11	5	1	19

V. ALGEBRAIC EQUATIONS

. Linear equations in one unknown with numerical coefficients	13w 14w	26 49w 48 34w	3 32abw	3c 15w	12
. Linear equations in one unknown with literal coefficients		44	11		
. Simple equations in one unknown, reducible to a linear equation			35	3ab	3
. Two linear equations in two unknowns with numerical coefficients - by elimination		16	14	1 17wr	4
. Two linear equations in two unknowns with numerical coefficients - by substitution		37		17wr	2
. Graphs of linear equations		25	12 19 26	2	5
Sub-Block Total	(2)	(8)	(9)	(7)	(26)
. Quadratic equations factored form		19		23w	2
. Quadratic equations - solution by factoring		39 47 50	24 43w	13	6
. Quadratic equations - solution by quadratic formula			45	5	2
. Graphs of quadratic equations			21	24	2
Sub-Block Total		(4)	(4)	(4)	(12)
Block Total	2	12	13	11	38

VI. INEQUALITIES

. Solution of a linear inequality in one unknown with numerical coefficients		36	5	4	3
. Solution of quadratic inequalities			34	20	2

	AR	EA	IA	PC	Total
. Graphing linear inequalities in one unknown		20			1
Block Total		2	2	2	6

VII. RATIONAL EXPRESSIONS

	AR	EA	IA	PC	Total
. Simplification of a rational expression by cancellation of common factors (1 and 2 variables)	28	10 30 33	6bc 29		7
. Evaluation of a rational expression (one and two variables)		27			1
. Addition and subtraction of rational expressions		40	17 22	8	4
. Multiplication and division of rational expressions		35	2 6a 39	9 11 16	7
Block Total	1	6	8	4	19

VIII. GEOMETRY

	AR	EA	IA	PC	Total
. Graph reading(interpretation)	6				1
. Points in the coordinate Plane	36	17	9		3
. Distance of two points in the coordinate plane			27		1
Sub-Block Total	(2)	(1)	(2)		(5)
. Measurement formulas for perimeter and area of triangles, squares, rectangles and parallelograms	38w	32 31w 41w			4
. Measurement formulas for circumference and area of circles	46w	46ac	41w		4
. Measurement formulas for volume of cubes, cylinders, rectangular solids, and spheres		46b			1
Sub-Block Total	(2)	(6)	(1)		(9)
. The sum of the interior angles of a triangle		7			1
. Properties of isosceles and equilateral triangles	39	7cr			2
. Properties of congruent and similar triangles	50	42		19w	3

	AR	EA	IA	PC	Total
. The Pythagorean theorem and special triangles (45-45-90, 30-60-90)		18			1
. Parallelism and perpendicularity					
Sub-Block Total	(2)	(5)		(1)	(8)
Block Total	7	12	3	1	23
IX. ABSOLUTE VALUE(AV)					
. Simplification and evaluation of expressions involving AV			8	12	2
. Solution of equation and evaluation of expressions involving AV				2	1
Block Total			1	2	3
X. TRIGONOMETRY					
. Right angle trigonometry				35	1
. Trigonometric functions as circular functions				30	1
. Radian and degree measure, special angles				36	1
. Graphs of trigonometric functions				39	1
. Use of trigonometric identities				37	1
Block Total				5	5
XI. FUNCTIONS					
. Function concept and use of function notation				21 28	2
. Function evaluation using substitution				26	1
. Compound function				27	1
. Graphs of functions including translations, reflections,and absolute value				32	1
Sub-Block Total				5	5
. Numerical function evaluation			15		1
. Literal function evaluation			25		1

	AR	EA	IA	PC	Total
. Definition, laws and rules of logarithms			20 40	31 33 40a	5
. Inverse relation between the logarithms and exponential function				38	1
. Logarithmic and exponential equations			37	40bc	3
. Graphs of logarithmic and exponential functions				29	1
			5	7	12
Block Total			5	12	17

XIII. PROBABILITY AND STATISTICS

. Probability

. Descriptive statistics

TABLE 2
THE DISTRIBUTION OF TEST ITEMS BY BLOCK AND SUB-BLOCK FROM THE MATHEMATICS DIAGNOSTIC TESTING PROJECT*

	AR	EA	IA	PC	Total
I. INTEGERS (3)	10 (3)	1			11
II. FRACTIONS, DECIMALS, PERCENT, RATIO, PROPORTION (13)	22(12)	8(7)	3(2)	1	34
A. Common Fractions (5)	12 (5)	2(2)			14
B. Decimal Fractions (3)	5 (3)	2(2)			7
C. Conversions (3)	2 (2)	2(1)			4
D. Ratio,Percentage and Proportion (2)	3 (2)	2(2)	3(2)	1	9
III. EXPONENTS, RADICALS AND SQUARE ROOTS (14)	6 (3)	8(6)	12(8)	5(5)	31
A. Exponents (3)	4 (2)	3(2)	4(3)	1	12
B. Square roots (4)	2 (1)	4(4)	1		7
C. Radical and Rational Expressions (7)		1	7(4)	4(4)	12
IV. POLYNOMIALS (12)	3 (2)	11(9)	5(5)	1	20
A. Arithmetic operations (8)	3 (2)	8(6)	3(3)	1	15
B. Factoring (4)		3(3)	2(2)		5

NOTES:

1. The test designations are as follows:

'AR': Algebra Readiness
'EA': Elementary Algebra
'IA': Intermediate Algebra
'PC': Precalculus

2. Numbers in parentheses are the number of subtopics included within the given topic for a given test. Note that greater differentiation was made for higher-level topics and algorithms.

	AR	EA	IA	PC	Total
V. ALGEBRAIC EQUATIONS (10)	2 (1)	12(7)	13(8)	11(9)	38
A. Linear equations (6)	2 (1)	8(4)	9(4)	7(4)	26
B. Quadratic equations (4)		4(2)	4(3)	4(4)	12
VI. INEQUALITIES (3)		2(2)	2(2)	2(2)	6
VII. RATIONAL EXPRESSIONS (4)	1	6(4)	8(3)	4(2)	19
VIII. GEOMETRY (11)	7 (6)	12(8)	3(3)	1	23
A. Coordinate plane (3)	2 (2)	1	2(2)		5
B. Measurement formulas (3)	2 (2)	6(3)	1		9
C. Properties of Triangles (5)	2 (2)	5(4)		1	8
IX. ABSOLUTE VALUE (2)			1	2(2)	3
X. TRIGONOMETRY (5)				5(5)	5
XI. FUNCTIONS (10)			5(4)	12(9)	17
A. Function concepts (4)				5(4)	5
B. Logarithmic and exponential Functions (6)			5(4)	7(4)	12
XIII. PROBABILITY AND STATISTICS					
TOTAL COUNTS (87)	51(28)	60(43)	52(36)	44(37)	207(8

NOTES (cont):

3. The Algebra Readiness Elementary Algebra and Intermediate Algebra Tests contained 50 items while the Precalculus test contained 40 items. There were 3 forms each of Elementary Algebra, Intermediate Algebra, and Precalculus; however, most items were common to all three forms.

TABLE 3
TOPICS OVERLAPS AMONG TESTS FROM THE MATHEMATICS DIAGNOSTIC TESTING PROJECT*

Test	AR	EA	IA	PC
Algebra Readiness (AR)	13	3	0	0
Elementary Algebra (EA)	12	15	1	0
Intermediate Algebra (IA)	7	12	14	2
Precalculus (PC)	4	9	10	13

NOTES:

1. There are 21 separate "topics" across all four tests at this level of specificity. This level corresponds to the subtopics listed in Table 2.

2. The diagonal elements above refer to the number of topics contained the specified MDTP test. Thus the Algebra readiness test contained items measuring 13 of the 21 topics at the level of specificity used here.

3. The numbers in the lower triangular portion of the table refer to the number of topics covered by both of the two specified MDTP tests. Thus there were 12 topics for which the Algebra Readiness and Elementary Algebra both contained items while there were only seven topics in common between the Algebra Readiness Test and the Intermediate Algebra Test.

4. The numbers in the upper triangular portion of the table refer to the number of topics uniquely covered by the two specified tests. Thus there were 3 topics contained only on both the Algebra Readiness and Elementary Algebra Tests but no topics for which only the Algebra Readiness and Intermediate Algebra Test contained items.

TABLE 4
CURRICULUM EXPECTATION STATEMENTS BY MDTP TOPICS

	NSF	CB	MC	F
I. INTEGERS				
. Basic operations with signed numbers			N1.1,2	J-N4;S-N1
. Prime factorization				J-N9
. Number line			N1.1	
II. FRACTIONS, DECIMALS, PERCENT, RATIO, PROPORTION				
. Equivalent fractions	10			
. Addition and subtraction of fractions				J-N2;S-N1
. Multiplication and division of fractions				J-N2;S-N1
. Order and comparison of fractions	10			J-N3
. Simplification of complex fractions				
. Addition and subtraction of decimals				J-N2;S-N1
. Multiplication and division of decimals				J-N2;S-N1
. Estimation and approximation with decimals	5			J-N1
. Conversion between fractions and decimal	10			J-N6
. Conversion between decimals and percent				J-N6

NOTES:

1. Entries in the table are the identification codes for the curriculum expectation statements presented in full in Appendix A.

2. An empty cell indicates that no element of the given curriculum statement matched the given MDPT topic.

3. Curriculum expectation statements are designated as follows:

NSF = Educating Americans for the 21st Century
CB = Academic Preparation in Mathematics: Teaching for Transition from High School to College
MC = Model Curriculum Standards, Grades 9-12, First Ed.
F = Mathematics Frameworks: For California Public Schools, Kindergarten Through Grade Twelve

	NSF	CB	MC	F
. Computations with both decimals and fractions, rounding off				
. Percentage				
. Proportion			A4.2	J-N6,7;S-A4
III. EXPONENTS, RADICALS AND SQUARE ROOTS				
. Laws of exponents			A7.1	J-N11
. Powers of 10 and scientific notation			N3.1	J-N10
. Exponentiation with integral exponents			N3.2;A1.2	J-N10;S-A5
. Square roots of perfect squares				J-N8
. Simplification of square roots				SA5
. Addition and subtraction of square roots		7		
. Multiplication and division of square roots		7		
. Conversion between radicals and exponent				
. Rationalization of the denominator				
. Addition and subtraction of radical expressions		7		
. Numerical calculations with rational exponents and radicals		7		SN3
. Algebraic calculations with rational exponents and radicals	27			SN3
. Factoring and simplifying an algebraic expression involving rational and literal exponents and radicals	15	8		
. Estimation and approximation with radica	5		A1.4	
IV. POLYNOMIALS				
. Arithmetic operations involving literal symbols	14,15		A1.1;A7.4	
. Simplification of a polynomial by grouping (1 and 2 variables)	15			J-A2

	NSF	CB	MC	F
. Addition and subtraction of polynomials	15		A7.3	S-A6
. Evaluation of a polynomial (1 and 2 variables)	15			J-A3
. Multiplication of a monomial with a polynomial	15			
. Multiplication of two binomials			A7.3	S-A6
. Division of polynomials	15			SA7
. Squaring a binomial (a+b)2, (a-b)2 and (a+b)(a-b)	26			
. Factoring polynomials by finding a common monomial factor	15		A7.2;A8.1	SA8
. Factoring a trinomial				
. Factoring perfect squares and differences of squares			A7.5	
. Complex numbers	30	11		
V. ALGEBRAIC EQUATIONS				
. Solution of a linear equation in one unknown with numerical coefficients	17	6	A2.1-4	J-A4;S-A1,3
. Solution of a linear equation in one unknown with literal coefficients	17	6		
. Solution of a simple equation in one unknown, reducible to a linear equation	17	6		
. Solution of two linear equations in two unknowns with numerical coefficients - by elimination	17	6	A6.3	S-A2
. Solution of two linear equations in two unknowns with numerical coefficients - by substitution	17	6	A6.3	
. Graphing a linear equation	19	23	A5.1	S-A2
. Solution of a quadratic equation from factored form	18	6		
. Solution of a quadratic equation by factoring	18	6		
. Solution of quadratic equations by quadratic formula	18	6		

	NSF	CB	MC	F
. Graphs of quadratic equations	19	23	A5.2	
VI. INEQUALITIES				
. Solution of a linear inequality in one unknown with numerical coefficients	17	6	A2.1	J-A5;S-A1,3
. Solution of quadratic inequalities	17	6		
. Graphing linear inequalities in one unknown				S-A2
VII. RATIONAL EXPRESSIONS				
. Simplification of a rational expression by cancellation of common factors (1 and 2 variables)				
. Evaluation of a rational expression (one and two variables)				
. Addition and subtraction of rational expressions				
. Multiplication and division of rational expressions				
VIII. GEOMETRY				
. Graph reading(interpretation)	36	1,19	G4.2	
. Points in the coordinate Plane	36	19		S-G7
. Distance of two points in the coordinate plane	36	19	G4.1	
. Measurement formulas for perimeter and area of triangles, squares, rectangles and parallelograms	11,36, 33	14	G1.3	J-M1
. Measurement formulas for circumference and area of circles	11,36, 33	14		J-M1
. Measurement formulas for volume of cubes, cylinders, rectangular solids, and spheres	11,36, 33	14	G1.3	J-M1
. The sum of the interior angles of a triangle	33	14	G1.1	J-G3,4; S-G2

	NSF	CB	MC	F
. Properties of isosceles and equilateral triangles	34	15	G3.1,2	J-G3,4; S-G2,6
. Properties of congruent and similar triangles	34	15	G3.1,2	J-G3,4; S-G3
. The Pythagorean theorem and special triangles (45-45-90, 30-60-90)	33,35, 36	14, 16	G5.1,2	J-G3,4; S-G2,5,6
IX. ABSOLUTE VALUE(AV)				
. Simplification and evaluation of expressions involving AV			A1.3	
. Solution of equation and evaluation of expressions involving AV				
X. FUNCTIONS				
. Function concept and use of function notation	13,21 28	22	P2.3	J-PF1
. Function evaluation using substitution	21	22	P2.1	J-PF1
. Compound function	21	22		
. Graphs of functions including translations, reflections,and absolute value	21,25	25	P2.2;A5.2	J-PF2; S-PF1,2
XI. TRIGONOMETRY				
. Right angle trigonometry			G6.1,2	
. Trigonometric functions as circular functions	24	24		
. Radian and degree measure, special angle				
. Graphs of trigonometric functions	25			
. Use of trigonometric identities				
XII. LOGARITHMIC AND EXPONENTIAL FUNCTIONS				
. Numerical function evaluation				
. Literal function evaluation				
. Definition, laws and rules of logarithms				

	NSF	CB	MC	F
. Inverse relation between the logarithms and exponential function	24	24		
. Logarithmic and exponential equations	27	10		
. Graphs of logarithmic and exponential functions	25			

XIII. PROBABILITY AND STATISTICS

	NSF	CB	MC	F
. Probability	7,20, 28	9,5	S2.1,2; S2.1,2,3	S-SP1,3,8; J-SP1-3,6
. Descriptive statistics	7,39	2	S3.1.2; S4.1-4	J-SP4,5; S-SP2,4,6,7

TABLE 5
SECOND INTERNATIONAL MATHEMATICS STUDY
INTERNATIONAL CONTENT GRID FOR GRADE 8
(SIMS ITEMS)

CONTENT CATEGORIES	COMPUTATION	COMPREHENSION	APPLIC/ ANALYS	Total
000 ARITHMETIC				
001 WHOLE NUMBERS	4	2	4	10
002 COMMON FRACTIONS	6	4	2	12
003 DECIMAL FRACTIONS	4	5	4	13
004 RATIO, PROPORTION, PERCENT	5	4	8	17
005 NUMBER THEORY	1	1	1	3
006 POWERS	3			3
008 SQUARE ROOTS		3		3
009 DIMENSIONAL ANALYSIS	1			1
	24	19	19	62
100 ALGEBRA				
101 INTEGERS	3	1	3	7
102 RATIONALS	1	2	1	4
103 INTEGER EXPONENTS	1	1		2
104 FORMULAS	3	3	3	9
105 POLYNOMIALS EXPRESSIONS	3			3
106 EQUATIONS AND INEQUATIONS	4	3	2	9
107 RELATIONS	2	1	1	4
110 FINITE SETS	1	1	2	4
	18	12	12	42
200 GEOMETRY				
201 CLASSIFICATION	1	2	2	5
202 PROPERTIES	2	2	5	9
203 CONGRUENCE	1	1	2	4
204 SIMILARITY	1	1	4	6
205 GEOMETRIC CONSTRUCTIONS			1	1
206 PYTHAGOREAN	2		1	3
207 COORDINATES	1	4	2	7
208 DEDUCTIONS		2	2	4
209 TRANSFORMATION (INFORMAL)	2	1	1	4
211 SOLIDS	1			1
212 SPATIAL VISUALIZATION		1	2	3
215 TRANSFORMATIONAL GEOMETRY	1	2	1	4
	12	16	23	51

300 PROBABILITY AND STATISTICS				
301 DATA COLLECTION		1		1
302 ORGANIZATION	1	2		3
303 REPRESENTATION		5	2	7
304 MEAN, MEDIAN, MODE	3	2	1	6
306 PROBABILITY	1			1
	----	----	----	----
	5	10	3	18
400 MEASUREMENT				
401 UNITS	3		2	5
402 ESTIMATION	1	2	3	6
403 APPROXIMATION		2	1	3
404 DETERMINING MEASURES	3	3	6	12
	----	----	----	----
	7	7	12	26
	====	====	====	====
	66	64	69	199

NOTES:

1. The content categories are those identified by the SIMS International Mathematics Committee. The numbers refer to the numbers of items that fall in each cell of the topic by cognitive level grid.

2. Students in 8th grade classes are refered to as Population A in the Second International Mathematics Study.

TABLE 6
SECOND INTERNATIONAL MATHEMATICS STUDY
INTERNATIONAL CONTENT GRID FOR GRADE 12
(SIMS ITEMS)

CONTENT CATEGORIES	COMPUTATION	COMPREHENSION	APPLIC/ ANALYS	Total
1 SETS, RELATIONS AND FUNCTIONS				
1.1 SET NOTATION				
1.2 SET OPERATIONS (E.G., UNION,INCLUSION)		2		2
1.3 RELATIONS				
1.4 FUNCTIONS				
1.5 INFINITE SETS, CARDINALITY AND CARDINAL ALGEBRA (RATIONALS AND REALS)	2	1	2	5
	----	----	----	----
	2	3	2	7
2 NUMBER SYSTEMS				
2.1 COMMON LAWS DOR NUMBER SYSTEMS	1	1	1	3
2.2 NATURAL NUMBERS		1	3	4
2.3 DECIMALS	1		2	3
2.4 REAL NUMBERS	1	1	1	3
2.5 COMPLEX NUMBERS	2	2	2	6
	----	----	----	----
	5	5	9	19
3 ALGEBRA				
3.1 POLYNOMIALS (OVER)	3	2	1	6
3.2 QUOTIENTS OF POLYNOMIALS	2			2
3.3 ROOTS AND RADICALS	2	1	2	5
3.4 EQUATIONS AND INEQUALITIES	1	2	3	6
3.5 SYSTEMS OF EQUATIONS AND INEQUALITIES	1	1	3	5
3.6 MATRICES			1	1
3.7 GROUPS, RINGS AND FIELDS				
	----	----	----	----
	9	6	10	25
4 GEOMETRY				
4.1 EUCLIDEAN (SYNTHETIC) GEOMETRY	1			1
4.2 AFFINE AND PROJECTIVE GEOMETRY IN THE PLANE				
4.3 ANALYTIC (COORDINATE) GEOMETRY IN THE PLANE	3	3	4	10
4.4 THREE-DIMENSIONAL COORDINATE GEOMETRY				

4.5	VECTOR METHODS	1	3	2	6
4.6	TRIGONOMETRY	2	1	3	6
4.7	FINITE GEOMETRIES				
4.8	ELEMENTS OF TOPOLOGY				
4.9	TRANSFORMATIONAL GEM	2	1	2	5
		----	----	----	----
		9	8	11	28
5	ANALYSIS				
5.1	ELEMENTARY FUNCTIONS	3	4	4	11
5.2	PROPERTIES OF FUNCTIONS		2	2	4
5.3	LIMITS AND CONTINUITY	1	2	1	5
5.4	DIFFERENTIATION	2	1	1	4
5.5	APPLICATIONS OF THE DERIVATIVE	5	1	4	10
5.6	INTEGRATION	2	2	2	6
5.7	TECHNIQUES OF INTEGRATION	1		1	2
5.8	APPLICATIONS OF INTEGRATION	2	2	1	5
5.9	DIFFERENTIAL EQUATIONS				
5.10	SEQUENCES AND SERIES OF FUNCTIONS				
		----	----	----	----
		16	19	16	46
6	PROBABILITY AND STATISTICS				
6.1	PROBABILITY	2		1	3
6.2	STATISTICS	2		1	3
6.3	DISTRIBUTIONS			1	1
6.4	STATISTICAL INFERENCE				
6.5	BIVARIATE STATISTICS				
		----	----	----	----
		4		3	7
7	FINITE MATHEMATICS				
7.1	COMBINATORICS		2	2	4
8	COMPUTER SCIENCE				
9	LOGIC				
		===	===	===	===
		45	38	53	136

NOTES:

1. Entries refer to number of items that fall in each cell of topic by cognitive level.

2. Grade 12 was labeled Population B in the Second International Math Study.

TABLE 7
SECOND INTERNATIONAL MATHEMATICS STUDY
INTERNATIONAL CONTENT GRID
(MDTP Items)

CONTENT CATEGORIES	COMPUTATION	COMPREHENSION	APPLIC/ANALYS
POPULATION A (8th Grade)			
000 ARITHMETIC			
001 WHOLE NUMBERS			AR(1)
002 COMMON FRACTIONS	AR(6)EA(1)	AR(2)EA(1)	EA(1)
003 DECIMAL FRACTIONS	AR(4)EA(2)	AR(2)EA(2)	
004 RATIO, PROPORTION, PERCENT	AR(1)EA(1)	AR(2)	AR(2)EA(1)
005 NUMBER THEORY			IA(3)PC(1)
006 POWERS	AR(4)EA(3)IA(1)		
008 SQUARE ROOTS	AR(2)EA(4)IA(1)	EA(1)	
009 DIMENSIONAL ANALYSIS			
100 ALGEBRA			
101 INTEGERS	AR(4)		AR(1)
102 RATIONALS			AR(1)
103 INTEGER EXPONENTS	IA(1)	IA(2)	
104 FORMULAS	AR(2)EA(2)IA(1)	EA(2)	AR(1)
105 POLYNOMIALS EXPRESSIONS	AR(2)EA(12) IA(9)PC(4)	IA(2)	EA(4)EA(7) PC(3)
106 EQUATIONS AND INEQUATIONS	EA(10)IA(7) PC(4)		AR(1)
107 RELATIONS			
110 FINITE SETS			
200 GEOMETRY			
201 CLASSIFICATION			
202 PROPERTIES	AR(2)EA(1)	EA(1)	EA(1)
203 CONGRUENCE			
204 SIMILARITY	AR(1)EA(1)PC(1)		
205 GEOMETRIC CONSTRUCTIONS			
206 PYTHAGOREAN	EA(1)		EA(1)
207 COORDINATES	IA(1)	EA(1)IA(1)	AR(1)
208 DEDUCTIONS			
209 TRANSFORMATION (INFORMAL)			
211 SOLIDS			
212 SPATIAL VISUALIZATION			
215 TRANSFORMATIONAL GEOMETRY			
300 PROBABILITY AND STATISTICS			
302 ORGANIZATION			

303 REPRESENTATION	AR(1)		
304 MEAN, MEDIAN, MODE			
306 PROBABILITY			
400 MEASUREMENT			
401 UNITS			AR(2)
402 ESTIMATION			
403 APPROXIMATION			
404 DETERMINING MEASURES	AR(2)EA(3)		EA(2)

POPULATION B

1 SETS, RELATIONS AND FUNCTIONS			
1.1 SET NOTATION			
1.2 SET OPERATIONS (E.G., UNION,INCLUSION)			
1.3 RELATIONS			
1.4 FUNCTIONS			
1.5 INFINITE SETS, CARDINALITY AND CARDINAL ALGEBRA (RATIONALS AND REALS)			
2 NUMBER SYSTEMS			
2.1 COMMON LAWS DOR NUMBER SYSTEMS			
2.2 NATURAL NUMBERS			
2.3 DECIMALS			
2.4 REAL NUMBERS	IA(1)		
2.5 COMPLEX NUMBERS			
3 ALGEBRA			
3.1 POLYNOMIALS (OVER)			
3.2 QUOTIENTS OF POLYNOMIALS			
3.3 ROOTS AND RADICALS	IA(6)	IA(2)	
3.4 EQUATIONS AND INEQUALITIES			
3.5 SYSTEMS OF EQUATIONS AND INEQUALITIES	IA(1)PC(1) EA(2)		PC(1)
3.6 MATRICES			
3.7 GROUPS, RINGS AND FIELDS			
4 GEOMETRY			
4.1 EUCLIDEAN (SYNTHETIC) GEOMETRY			
4.2 AFFINE AND PROJECTIVE GEOMETRY IN THE PLANE			
4.3 ANALYTIC (COORDINATE) GEOMETRY IN THE PLANE			
4.4 THREE-DIMENSIONAL			

	COORDINATE GEOMETRY			
4.5	VECTOR METHODS			
4.6	TRIGONOMETRY	PC(4)		PC(1)
4.7	FINITE GEOMETRIES			
4.8	ELEMENTS OF TOPOLOGY			
4.9	TRANSFORMATIONAL GEM			
5	ANALYSIS			
5.1	ELEMENTARY FUNCTIONS	IA(3)PC(5)	IA(2)PC(6)	PC(2)
5.2	PROPERTIES OF FUNCTIONS			
5.3	LIMITS AND CONTINUITY			
5.4	DIFFERENTIATION			
5.5	APPLICATIONS OF THE DERIVATIVE			
5.6	INTEGRATION			
5.7	TECHNIQUES OF INTEGRATION			
5.8	APPLICATIONS OF INTEGRATION			
5.9	DIFFERENTIAL EQUATIONS			
5.10	SEQUENCES AND SERIES OF FUNCTIONS			
6	PROBABILITY AND STATISTICS			
6.1	PROBABILITY			
6.2	STATISTICS			
6.3	DISTRIBUTIONS			
6.4	STATISTICAL INFERENCE			
6.5	BIVARIATE STATISTICS			
7	FINITE MATHEMATICS			
7.1	COMBINATORICS			
8	COMPUTER SCIENCE			
9	LOGIC			

NOTES

1. Entries in this table refer to the number of items on each of the 4 MDTP test that fall in each topic-by-cognitive-level cell.

2. MDTP tests are designated as follows:

AR = Algebra Readiness
EA = Elementary Algebra
IA = Intermediate Algebra
PC = Pre-Calculus

3. There were two absolute value items on the MDTP intermediate algebra test that we wre unable to classify in the SIMS grid.

TABLE 8
SECOND INTERNATIONAL MATHEMATICS STUDY
INTERNATIONAL CONTENT GRID FOR Grade 8
(AR MDTP Items)

CONTENT CATEGORIES	COMPUTATION	COMPREHENSION	APPLIC/ANALYS
000 ARITHMETIC			
001 WHOLE NUMBERS			22
002 COMMON FRACTIONS	2,11,18, 31,42,43,46	24,37	
003 DECIMAL FRACTIONS	1,3,17,25	32,41	
004 RATIO, PROPORTION, PERCENT	10	26,30	19,44
005 NUMBER THEORY		12,20	
006 POWERS	29,34,40, 47		
008 SQUARE ROOTS	23,48		
009 DIMENSIONAL ANALYSIS			
100 ALGEBRA			
101 INTEGERS	7,9,21,27		
102 RATIONALS			35
103 INTEGER EXPONENTS			16
104 FORMULAS	4,15		13
105 POLYNOMIALS EXPRESSIONS	28,33		
106 EQUATIONS AND INEQUATIONS			
107 RELATIONS			14
110 FINITE SETS			
200 GEOMETRY			
201 CLASSIFICATION			
202 PROPERTIES	39,46		
203 CONGRUENCE			
204 SIMILARITY	50		
205 GEOMETRIC CONSTRUCTIONS			
206 PYTHAGOREAN			
207 COORDINATES			36
208 DEDUCTIONS			
209 TRANSFORMATION (INFORMAL			
211 SOLIDS			
212 SPATIAL VISUALIZATION			
215 TRANSFORMATIONAL GEOMETRY			
300 PROBABILITY AND STATISTICS			
302 ORGANIZATION			
303 REPRESENTATION	6		
304 MEAN, MEDIAN, MODE			
306 PROBABILITY			
400 MEASUREMENT			
401 UNITS			5,8
402 ESTIMATION			
403 APPROXIMATION			
404 DETERMINING MEASURES	38,49		

TABLE 9
SECOND INTERNATIONAL MATHEMATICS STUDY
INTERNATIONAL CONTENT GRID FOR GRADE 8
(EA MDT Items)

CONTENT CATEGORIES	COMPUTATION	COMPREHENSION	APPLIC/ANALYS
000 ARITHMETIC			
001 WHOLE NUMBERS			
002 COMMON FRACTIONS	15	43	21
003 DECIMAL FRACTIONS	2b,5	2a,c	
004 RATIO, PROPORTION PERCENT	29		45w
005 NUMBER THEORY			
006 POWERS	4,8,38		
008 SQUARE ROOTS	9,11,24,28	3	
009 DIMENSIONAL ANALYSIS			
100 ALGEBRA			
101 INTEGERS			
102 RATIONALS			
103 INTEGER EXPONENTS			
104 FORMULAS	1,22		
105 POLYNOMIALS EXPRESSIONS	14,12a,b,c, 23b,10,30,27, 35,40	13,23a,c	
106 EQUATIONS AND INEQUATIONS	26,44,48, 16,37		34,25,20,49
107 RELATIONS			
110 FINITE SETS			
200 GEOMETRY			
201 CLASSIFICATION			
202 PROPERTIES	7	7c	32
203 CONGRUENCE			
204 SIMILARITY	42		
205 GEOMETRIC CONSTRUCTIONS			
206 PYTHAGOREAN	18		32r
207 COORDINATES		17	
208 DEDUCTIONS			
209 TRANSFORMATION (INFORMAL)			
211 SOLIDS			
212 SPATIAL VISUALIZATION			
215 TRANSFORMATIONAL GEOMETRY			
300 PROBABILITY AND STATISTICS			
302 ORGANIZATION			
303 REPRESENTATION			
304 MEAN, MEDIAN, MODE			
306 PROBABILITY			
400 MEASUREMENT			
401 UNITS			
402 ESTIMATION			
403 APPROXIMATION			
404 DETERMINING MEASURES	46a,b,c		31,41

NOTES

1. Grade 8 was labeled Population A in the Second International Math Study.

2. The numbers refer to the item numbers from specific MDTP forms. When an attached letter follwos, the itme appeared only on that specific MDTP form. Otherwise it appeared on all three alternate forms.

TABLE 10
GRADE 8 MATHEMATICS DUPLEX DESIGN
(MDTP Items)

Content Categories	a. Procedural Skills	b. Factual Knowledge	c. Higher Level Thinking
10. Numbers			
Integers	AR(8)		AR(4)
Fractions	AR(7),EA(4)	AR(4)	AR(2),EA(2), IA(1),PC(1)
Percent	AR(2),EA(1)		AR(2),IA(1)
Decimals	AR(7),EA(4)		EA(1),IA(1)
Irrationals	EA(3),IA(1)	AR(2),EA(1)	
Exponents?			
20. Algebra			
Expressions	AR(4),EA(17), IA(14),PC(6)		
Equations	EA(8),IA(6), PC(6)		AR(2),EA(3), IA(7),PC(5)
Inequalities	EA(1),IA(2) PC(3)		EA(1)
Functions	IA(4),PC(9)	IA(1)	PC(3)
30. Geometry			
Figures		IA(1)	
Relations & Transformations	EA(2),IA(2)	IA(1)	
Coordinates	IA(2)	AR(2),EA(1)	AR(1)
40. Measurement			
English & metric units			
Length, area & volume	AR(1)	AR(1),EA(2)	AR(1),EA(2)
Other systems (time,distance)			

50. Probability & Statistics

Probability

Experiments & surveys

Descriptive statistics

NOTES:

1. Cognitive levels are as follows:

Procedural Skills-- Calculating, Rewriting, Constructing, Estimating, Executing Algorithms

Factual Knowledge -- Terms, Definitions, Concepts

Higher Level Thinking -- Proof, Reasoning, Problem Solving, Real-world applications

MDTP Items Not Classified: 9 -- Exponents, Trigonometry, Radicals

2. The MDTP tests are designated as follows:

AP = Algebra Readiness
EA = Elementary Algebra
IA = Intermediate Algebra
PC = Pre-Calculus

3. Entries indicate the number of MDPT items on each test that fall in each topi-by-cognitive-level cell of the DUPLEX design.

TABLE 11

DISTRIBUTION OF SECOND INTERNATIONAL MATHEMATICS STUDY TEST ITEMS POPULATION A (GRADE 8) AMONG THE CELLS OF THE GRADE 8 MATHEMATICS DUPLEX DESIGN MATRIX

Content Categories	SIMS Grid Categories	a. Procedural Skills	b. Factual Knowledge	c. Higher Level Thinking
10. Numbers				
Integers	001,101 403	8	3	7
Fractions	002,102	6	5	2
Percent	004	5	4	8
Decimals	003	4	5	4
Irrationals	008	-	3	-
20. Algebra				
Expressions	102,104 105	7	4	4
Equations	106	3	1	2
Inequalities	106	1	2	-
Functions	107	2	1	1
30. Geometry				
Figures	201,208 211-212	3	3	5
Relations & Transformations	202-206 208,209 215	9	9	16
Coordinates	207	1	4	2
40. Measurement				
English & Metric Units	401	3	-	2
Length, Area & Volume	402-404	10	3	6
Other systems (time, distance)	009	1		

50. Probability & Statistics				
Probability	306	1	-	-
Experiments & Surveys	301-303	1	8	2
Descriptive Statistics	304	3	2	1

NOTES:

1. The definitions of the process categories provided by Bock and Mislevy are as follows:

   Procedural Skills-- Calculating, Rewriting, Constructing, Estimating, Executing Algorithms

   Factual Knowledge -- Terms, Definitions, Concepts

   Higher Level Thinking -- Proof, Reasoning, Problem Solving, Real-world applications

2. The numbers indicated as SIMS Grid Categories are according to grid numbering scheme reported in several of the SIMS reports (e.g.,Chang & Ruzicka, 1985). The classification of the items into cells of that SIMS grid corresponds to decisions by the International Mathematics Committee as reported in the SIMS longitudinal data bank.

3. We were unable to classify certain categories of SIMS items into the Duplex Design classification matrix. The categories we were unable to classify were as follows:
   - 005 Number Theory
   - 006 Powers
   - 103 Integer Exponents
   - 110 Finite Sets

   Certain SIMS categories (102, 106, 208, 403) contained items that belong to multiple cells from the Duplex Design matrix.

TABLE 12
Distribution of Test Items from the California Assessment Program Survey of Academic Skills: Grade 8 Mathematics

(Total number of questions: 468)

	Percent
I. Numbers (72)	15.4
A. Skills/concepts (50)	10
1. Order relations and classification (15)	3
2. Number theory (20)	4
3. Properties (15)	3
B. Applications (22)	5
II. Operations (72)	15.4
A. Skills/concepts (36)	8
1. Whole and rational numbers (22)	5
2. Percents (14)	3
B. Applications (36)	8
1. One-step (20)	4
2. Two or more steps (16)	4
III. Algebra (72)	15.4
A. Skills/concepts (50)	11
1. Expressions, equations, & inequalities (30)	6
2. Graphs and functions (20)	5
B. Applications (22)	5
IV. Geometry (72)	15.4
A. Skills/concepts (47)	10
1. Geometric terms and figures (20)	4
2. Relationships (25)	6
B. Applications (25)	6
V. Measurement (42)	9.0
A. Skills/concepts (24)	6
1. Units and estimates (12)	3
2. Perimeter, area, and volume (12)	3
B. Applications (18)	3

VI. Probability and Statistics (36)	7.7
A. Probability (18)	4
B. Statistics (18)	4
VII. Tables, Graphs, & Integrated Applications (30)	6.4
A. Tables and graphs (15)	3
B. Integrated applications (15)	3
VIII. Problem Solving (72)	15.4
A. Formulations of problem (14)	3
B. Analysis of problem (20)	4
C. Applying strategies (24)	5
D. Reasoning and interpretation (14)	3
E. Solution of problems (aggregations of all applications, probability and statistics, tables, graphs, & integrated applications)(261)	41

NOTE:

Entries in parentheses are number of items.

TABLE 13
Distribution of Test Items from the California Assessment
Program Survey of Basic Skills: Grade 12
Mathematics 1985-1986

(Total number of questions: 198)

	Percent
I. Arithmetic (98)	49.5
A. Number concepts (28)	14
1. Number and numeration (14)	7
2. Number theory (8)	4
3. Properties (5)	3
B. Whole numbers (22)	11
1. Computation (14)	7
2. Application (8)	4
C. Fractions (26)	13
1. Computation (14)	7
2. Application (12)	6
D. Decimals (22)	11
1. Computation (14)	7
2. Application (8)	4
II. Algebra (32)	16
A. Computation (14)	7
B. Applications (18)	9
III. Geometry (24)	12
A. Knowledge of facts (12)	6
B. Applications (12)	6
IV. Measurement (30)	15
A. Knowledge of facts (12)	6
B. Applications (18)	9
V. Probability and Statistics (14)	7
A. Computation (6)	3
B. Application (8)	4
VII. All application problems (62)	31
A. Arithmetic (28)	14
B. Graphs (34)	17

NOTE:

Entries in parentheses are number of items.

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APPENDIX A

Curriculum Expectations in
Educating Americans for 21st Century
(Washington: National Science Foundation, 1983, pp.93-96)

K-8 Mathematics

In general changes de-emphasize excess drill in paper and pencil computations and emphasize developing better number sense.

*1. Understanding of arithmetic operations and knowledge of when and where specific operations should be used.

*2. Development of a thorough understanding of and facility with one digit number facts.

*3. Ability to use selectively calculators and computers to help develop concepts and to do many of the tedious computation that previously had to be done by paper and pencil.

*4. Development of skill in the use of informal mental arithmetic, first in providing exact answers to simple problems and later, approximate aswers to more complicated problems.

+5. Development of skills in estimation and approximation.

6. Development of problem-solving abilities. Trial and error methods, guessing and guestimating in solving word problem should be actively encouraged at all levels.

7. Understanding elementary data analysis, elementary statistics, and probability.

*8. Knowledge of place value, decimals, percent, and scientific notation.

*9. Understanding the relationship of numbers to geometry.

10. Understanding of fractions as numbers, comparisons of fractions, and conversion to decimals.

+11. Development of an intuitive geometric understanding and ability to use the mensuration formulas for two and three-dimensional figures.

*12. Ability to use the concepts of sets and some of the language of sets where appropriate. However, sets and set language are useful tools, not end goals, and it is inappropriate to start every year's program with a chapter on sets.

13. Understanding of elementary function concepts including dynamic models of increasing and decreasing phenomena.

14. Ability to use some algebraic symbolism and techniques,

particularly in grades 7-8.

Secondary School Mathematics

Streamline traditional components leaving room for important new topics. Give serious consideration to the development of an integrated secondary school mathematics curriculum.

Algebra

15. Ability to recognize basic algebraic forms and know how to transform them into other forms.

*16. Understanding of the logic behind algebraic manipulations.

17. Skill in solving linear equations and inequalities.

18. Skill in solving quadratic equations.

19. Ability to graph linear and simple quadratic functions and use these in interpreting and solving problems.

20. Familiarity with permutations, combinations, and simple counting problems.

21. Knowledge of relations and functions.

22. Development of problem-solving abilities.

(For talented math students)

*23. Skill in solving higher degree equations and inequalities.

24. Knowledge of various types of functions including polynomial, exponential, logarithmic, and circular functions.

25. Ability to graph higher degree functions.

26. Familiarity with Binomial Theorem.

+27. Skill in solving trigonometric, exponential, and logarithmic equations.

28. Familiarity with arithmetic and geometric sequences and series.

*29. Knowledge of simple matrix operations and their relation to systems of linear equations.

30. Skill in operations with complex numbers.

Geometry

Introduce geometry together with algebra in 8th and 9th grades.

*31. Ability to think logically.

*32. Ability to work through short sequences of rigorously developed material while de-emphasizing column proof.

33. Knowledge of two- and three-dimensional figures and their properties.

+34. Ability to think in two and three dimensions in terms of congruence and similarity.

35. Ability to use the Pythagorean theorem and special right triangle relationships.

+36. Understanding of algebraic methods in geometry and analytic geometry; and vector algebra, especially in three dimensions.

*37. familiarity with computer graphics packages to get a visual sense of geometric concepts and transformation.

Other Mathematics

+38. Knowledge of discrete mathematics (basic combinatorics, graph theory, discrete probability).

39. Understanding of elementary statistics (data analysis, interpretation of tables, graphs, surveys, sampling).

*40. Knowledge of computer science (programming, introduction to algorithms and iteration).

*41. Familiarity with the philosophical basis of calculus and understanding of the elementary concepts of the subject (e.g., rates of change, intuitive ideas of limits).

NOTES

'*' indicates that the item was not found in the tests.

'+' indicates that only part of the item was found in the tests.

APPENDIX B

Curriculum Expectation Statements in Academic Preparation in Mathematics: Teaching for Transition from High School to College
(College Entrance Examination Board, 1985)

Statistics

+ 1 The ability to gather and interpret data and to represent them graphically.

2 The ability to apply techniques for summarizing data, using such statistical concepts as average, median, and mode.

* 3 Familiarity with techniques of statistical reasoning and common misuses of statistics.

* 4 Understanding of simulation techniques used to model experimental situations.

5 Knowledge of elementary concepts of probability needed in the study and understanding of statistics.

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Algebra

6 Skill in solving equations and inequalities.

7 Skill in operations with real numbers.

8 Skill in simplifying algebraic expressions, including simple rational and radical expressions

9 Familiarity with permutations, combinations, simple counting problems, and the binomial theorem.

Those students who will take advanced mathematics in college will need additional preparation in high school, including:

10 Skill in solving trigonometric, exponential, and logarithmic equations.

11 Skill in operations with complex numbers.

* 12 Familiarity with arithmetic and geometric series and with proofs by mathematical induction.

* 13 Familiarity with simple matrix operations and their relation to systems of linear equations.

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Geometry

14 Knowledge of two-dimensional and three-dimensional figures and their properties.

15 The ability to think of two-dimensional and three-dimensional figures in terms of symmetry, congruence, and similarity.

16 The ability to use the Pythagorean theorem and special right-triangle relationships.

17 The ability to draw geometrical figures and use geometrical modes of thinking in solving problems

Those students who plan to enter fields that will require advanced mathematics courses in college will need additional work in geometry beyond the topics listed above. They will need at least the following:

* 18 Appreciation of the role of proofs and axiomatic structure in mathematics and the ability to write proofs.

19 Knowledge of analytic geometry in the plane.

* 20 Knowledge of the conic sections.

* 21 Familiarity with vectors and with the use of polar coordinates.

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Functions

22 Knowledge of relations, functions, and inverses.

23 The ability to graph linear and quadratic functions and use them in the interpretations and solution of problems

24 Knowledge of various types of functions including polynomial, exponential, logarithmic, and circular functions.

25 The ability to graph such functions and to use them in the solution of problems.

'*' indicates that the item was not found on the tests.

'+' indicates that only part of the item was found on the tests.

APPENDIX C

Curriculum Expectations Statements in
Model Curriculum Standards, Grades 9-12, First Ed.
(California State Board of Education, 1985, pp. M-6 - M-41)

Number: Concepts and Skills

N1. Demonstrate an understanding of signed numbers

N1.1 Use the number line to demonstrate addition and subtraction of signed numbers.

N1.2 Multiplication and division of signed numbers.

* N1.3 Explain the value and uses of negative numbers.

N2. Students extend their undertanding of the real number system to include irrational numbers.

* N2.1 Irrational numbers and the need for them in the number system.

* N2.2 Locate real numbers on the number line.

N3. Students understand and are able to use integral exponents.

N3.1 Mental arithmetic to perform multiplication and divisions involving powers of ten.

N3.2 Evaluate expressions with rational bases.

Measurement: Concepts and Skills

M1. Students understand and use skills and concepts of measurement

* M1.1 Understand and apply the relationship between the precision of measurements and the accuracy of a calculation based on the measurements.

* M1.2 Make conversions within a measurement system.

* M1.3 Select and use appropriate formulas and procedures to determine a measure when a direct measurement tool is not available.

Geometry: Concepts and Skills

G1. Students understand the basic postulates, theorems, and definitions of geometry.

G1.1 Understand and can apply the basic postulates, theorems, and definitions of Euclidean geometry.

G1.2 Solve problems relating to special polygons such as triangles, parallelograms, rectangles, and squares.

G1.3 Use formulas to determine the lateral area, total area, and volume of certain three-dimensional figures such as cubes, rectangular solids, spheres, cones, pyramids, and cylinders.

G2. Students use compas and staightedge to perform geometric constructions.

* G2.1 Perform the basic geometric constructions.

* G2.2 Use the basic constructions to perform other specified constructions.

G3. Students use transformations to illustrate congruence and similarity of figures and ratio and proportion to illustrate similarity.

G3.1 Derive, intuatively, the conditionsnecessary for congruance.

G3.2 Derive, intuitively, the conditions necessary for similarity.

G4. Students understand and are able to use the basic elements of coordinate geometry.

G4.1 Use the distance formula to find the distance between two points.

G4.2 Derive information from the graph of a line.

G5. Students understand and are able to use the Pythagorean theorem.

G5.1 Follow and understand informal proof of the Pythagorean theorem.

G5.2 Explore right triangles by applications of the Pythagorean teorem and its converse.

G6. Students understand and are able to apply the sine, cosine, and tangent ratios in given right triangles.

G6.1 Understand the sine, cosine, and tangent ratios.

G6.2 Use the trigonometric ratios to solve right trangles.

G7. Students are able to visualize three-dimensional objects based on two-dimensional representations.

* G7.1 Construct three-dimensional models from two-dimensional

patterns.

- * G7.2 Construct patterms for polyhedra models.
- * G7.3 Explore isometric and orthographic representations of three-dimensional objects.
- * G7.4 Locate points in relations to x, y, and z axes.

Patterns and Functions concepts and Skills

P1. Students determine and extend patterns.

- * P1.1 Find patterns in sequences of numbers.
- * P1.2 Find patterns in the properties of geometric figures.

P2. Students understand the functional relationship between two variables.

- P2.1 Solve probelms involving direct and inverse variation.
- P2.2 Graph a function based on the inoformation given in a table or other nonalgebraic form.
- P2.3 Investigate various properties of functions.

Statistics and Probability: Concepts and Skills

S1. Students use counting procedures to solve combinatorial problems.

- S1.1 Use of list or a tree diagram to count possible arrangements.
- S1.2 Calculate posible combinations and use the multiplication principle.

S2. Students understand and use certain princples of probability.

- S2.1 Determine a sample space to represent the outcomes of an experiment.
- S2.2 Assign probabilities to elements of a sample space and calculate probabilities.
- S2.3 Distinguish between dependent and independent events and use conditional probability.

S3. Students determine measures of central tendency and dispersion of data they have collected.

S3.1 Construct a frequency table.

S3.2 Calculate median, mean, mode, quartiles, and standard deviation.

S4. Students interpret data and make valid inferences regarding the data.

S4.1 Explain the significance of differing values of statistical measures.

S4.2 Choose appropriate statistical measures to describe data.

S4.3 Identify and explain misuses of statistical data.

S4.4 Estimate probabilities of events based on empirical data and use these probabilities to make inferences.

Logic: Concepts and Skills

L1. Students understand and use certain terms and priniples of logical inference.

* L1.1 Distinguish between inductive and deductive reasoning and explain when each is appropriate.

* L1.2 Use inductive reasoning.

* L1.3 Use deductive reasoning in reaching conclusions.

Algebra: Concepts and Skills

A1. Students evaluate algebraic expressions when rational numbers are substituted for the variables.

A1.1 Apply the rules for the order of operations to evaluate an expression.

A1.2 Evaluate expressions involving exponents.

A1.3 Apply the definition of absolute value to evaluate expressions.

A1.4 Use estimation to give rational approximations of square roots.

A2. Student solve for a vaiable in equations or inequalities involving one or more variables.

A2.1 Solve a linear equation or enequality in one varible.

A2.2 Solve a formula for an indicated vaiable in the first degree.

A2.3 Solve practical problems involving direct and inverse variation.

A2.4 Apply the appropriate formulas to determine area, density, distance, and so on.

A3. Students apply algebraic techniques in solving word problems.

A3.1 Use variables to represent unknown quantities and write equations or inequalities.

A3.2 Formulate a problem given the representation of a variable and an equation or enequality.

A4. Students understand and can apply the concepts of ratio, proportion, and percent.

A4.1 Set up and solve proportions for a variety of situations.

A4.2 Apply formulas to solve practical problems involving percent.

A5. Students graph and/or analyze a variety of algebraic relations.

A5.1 Graph a linear equation or inequality in two variables.

A5.2 Graph and/or analyze the graph of a quadratic function or relation and identify its characteristics.

A6. Students solve certain systems of equations and inequalities.

* A6.1 Graph a system of two linear equations in two variables and interpret the meaning of the graph.

A6.3 Use algebraic techniques to solve a system of two linear equations.

A7. Students convert algebraic expressions to desired forms.

A7.1 Apply the rules for exponents to rewrite algebraic expressions.

A7.2 Simpligy algebraic fractions by dividing out the reates common monomial factor.

A7.3 Perform the operations of addition, subtraction, and multiplication on two binomials.

* A7.4 Perform the four basic operations on simple rational expressions involving monomial numerators and denominators.

A7.5 Find approximate values for expressions involving square roots.

A8. Students recognize special types of polynomials and are able to factor certain algebraic expressions.

A8.1 Factor polynomials by removing the greatest common factor.

* A8.2 Investigate ways in which factoring and other techniques can simplify mental calculations.

NOTES

'*' indicates that the item was not found on the tests

APPENDIX D

Curriculum Expectations in
Mathematics Frameworks: for California Public Schools Kindergarten Through Grade Twelve

(Sacramento, CA: State Department of Education pg. 31 - 32)

[Junior Program]

Number

JN1. Estimate answers to all computational problems. For a given problem, the student should be able to decide whether a proposed numerical answer could be a possible solution.

JN2. Find the sum, difference, product, or quotient of whole numbers, fractions, and decimals, choosing an appropriate method of calculation - estimation, mental arithmetic, paper and pencil, or the calculator - according to the nature of the problem.

JN3. Use the order relations to compare whole numbers, fractions, and decimals; and locate these numbers on the number line.

JN4. Understand and perform basic operations on the full set of rational numbers - positive, negative, and zero; develop an awareness of the underlying structure on which these operations are based; and recognize the various forms for representing rational numbers.

JN5. Use the associative and commutative laws of addition and multiplication and the distributive law to simplify numerical calculations.

JN6. Convert among percent, fractional, and decimal equivalents; and solve for the unknown in a percent problem.

JN7. Use ratios and proportions to solve a variety of problems, including those involving percent.

+JN8. Find the square roots of whole numbers, fractions, and decimals whose square roots are rational; and estimate irrational square roots as a check of results obtained with calculators.

JN9. Find the prime factors of whole numbers; and use prime factorization to find factors, multiples, greatest common factors, and least common multiples of a set of whole numbers.

JN10. Evaluate numerical expressions involving positive integral exponents; represent the prime factorization of a whole number in exponential notation; and convert whole numbers from base-ten notation to scientific notation and vice-versa.

JN11. Use the additive and multiplicative laws of exponents to simplify arithmetic expressions involving positive integral exponents and to multiply and raise numbers represented in scientific notation to powers.

Measurement

JM1. Choose an appropriate unit of measure and use the appropriate formulas to find the perimeter, circumference, and area of polygons and circles, the volume and surface area of selected solids, and the measures of angles.

*JM2. Convert units of measure of length, area, and volume within a system of measurement.

*JM3. Find perimeter, area, surface area, and volume of irregular geometric figures.

*JM4. Explore the approximate nature of measurement and the degree of error, using the concept of rounding.

*JM5. Use a ruler and protractor with proficiency.

Geometry

*JG1. Use and give examples to represent geometric terms, such as acute, right, obtuse, complementary, supplementary, and vertical angles; parallel, perpendicular, and intersecting lines and planes; and measures of angles. Identify two- and three-dimensional geometric figures.

JG2. Describe relationships between figures (congruent, similar) and perform transformations (rotations, reflections, translations, and dilations).

JG3. Demonstrate relationships involving geometric elements (for example, the sum of the measures of the angles of a triangle -- 180 degrees; two points needed to determine a line; three noncollinear points needed to determine a plane; the Pythagorean theorem; symmetry about a point and a line; supplementary and complementary angles; scalene, isosceles, equilateral, and right triangles).

JG4. Use knowledge of geometric terms, figures, or relationships to solve problems.

*JG5. Construct geometric figures, using compass and straightedge.

Patterns and Functions

JPF1. Determine the function rule that represents a relationship and find the value of the function for a given value of the variable.

JPF2. Graph simple relations and functions in all quadrants of the coordinate plane.

*JPF3. Represent functions in several ways: with a graph, as ordered pairs of numbers, in a verbal statement, or as an algebraic rule.

Statistics and Probability

JSP1. Represent the probability of an event as a fraction.

JSP2. Find the empirical probability of an event from a sample of observed outcomes.

JSP3. Find the probability of complementary events and of mutually exclusive events.

JSP4. Generate a frequency distribution for a given list of data; and compute the mean, median, mode, and range.

JSP5. Extract valid information from graphs, tables, and schedules.

JSP6. Use a list or tree diagram to count permutations or combinations.

Logic

*JL1. Explore the notion of mathematical implication.

*JL2. Explore the meaning of the logical connectives and, or, if ...then, and not.

*JL3. Determine the validity of simple arguments.

*JL4. Determine the equivalence of logical expressions.

*JL5. Perform simple deductive and inductive reasoning exercises.

Algebra

*JA1. Translate English phrases and sentences into algebraic expressions and vice versa.

JA2. Simplify algebraic expressions.

JA3. Substitute numerical and algebraic terms for variables in algebraic expressions; and use the standard order of operations to evaluate these expressions.

JA4. Solve linear equations of the form $ax + b = c$, using integers, fractions, and decimals.

JA5. Solve simple inequalities; and display solution sets on the number line.

*JA6. Represent mathematical patterns, using variables.

pg. 36 - 42

PROGRAM CONTENT

The SENIOR high school program in mathematics should enable students to do the following:

Number

SN1. Use operations on positive and negative rational numbers, including fractions and decimals, to determine sums, differences, products, and quotients quickly and accurately.

*SN2. Select and use the symbols of equality and inequality, operational symbols, and properties of the number system to write mathematical expressions and sentences that satisfy given conditions.

SN3. Evaluate formulas and numerical expressions involving powers and roots, grouping symbols, and the rules for the order of operations to solve a variety of applied problems.

*SN4. Locate integers on a number line and approximate the location of rational and irrational numbers to demonstrate the concept of ordering.

Measurement

*SM1. Understand and apply the relationships between the precision of measurements and the accuracy of a calculation based on the measurements.

*SM2. Make conversions within measurement systems, using conversion tables and equivalence of units.

*SM3. Select and use appropriate formulas and procedures to determine a measure when a direct measurement tool is not available.

Geometry

*SG1. Use the terminology of geometry, including terms that describe angles, lines, polygons, circles, and three dimensional figures: cubes, cones cylinders, prisms, pyramids, and spheres.

SG2. Understand and apply the basic postulates, definitions, and theorems of Euclidean geometry.

SG3. Employ the concepts of reflection, rotation, and translation to demonstrate symmetry and congruence of figures. Use ratio and proportion to demonstrate symmetry and congruence of figures. Use ratio and proportion to demonstrate similarity.

*SG4. Perform standard geometric constructions with a compass and a straightedge.

SG5. Apply the Pythagorean theorem and right-triangle trigonometry in practical problems.

SG6. Solve simple algebraic problems involving properties of polygons, including special quadrilaterals - square, rhombus, rectangles, parallelogram, and trapezoid - and special triangles - isosceles, equilateral, and right (including 30-60-90, 45-45-90 degrees).

+SG7. Apply the principles of coordinate geometry to graph lines, determine the slope and intercept of a line and the midpoint of a line segment, and determine the areas of special geometric figures.

Patterns and Functions

SPF1.Identify functions and inverse functions by inspection of their graphs.

SPF2.Translate graphs vertically and horizontally.

*SPF3.Graph linear inequalities in two variables.

*SPF4.Graph nonlinear functions that represent practical situations and interpret the graphs.

Statistics and Probability

SSP1.Use counting procedures to solve combinatorial problems.

SSP2.Develop an understanding of the common mathematical properties of the mean of a set of data.

SSP3.Use mathematical expectation to make judgements about the possible outcomes of random events.

SSP4.Explain the significance of varying values of statistical measures.

SSP6.Choose appropriate statistical measures to describe data.

SSP7.Identify and explain misuses of statistical measures.

SSP8.Estimate probabilities of events based on empirical data and use these results to make inferences.

Logic

*SL1. Distinguish between inductive and deductive reasoning and explain when each is appropriate.

*SL2. Use inductive reasoning to generate hypotheses.

*SL3. Use deductive reasoning to reach conclusions.

*SL4. Recognize when the conditions of a definition are met.

*SL5. Identify the distinction between a necessary condition and a sufficient condition. Explain what is meant by a necessary and sufficient condition.

*SL6. Recognize and explain flaws in invalid arguments.

Algebra

SA1. Solve a linear equation or inequality in one variable and explain the steps used.

SA2. Formulate and solve systems of linear equations or inequalities algebraically or graphically.

SA3. Solve practical problems involving direct and inverse variation.

SA4. Solve practical problems involving ratio, proportion, and percent.

SA5. Simplify and evaluate algebraic expressions involving positive and negative integral exponents and square roots.

SA6. Perform the operations of addition, subtraction, and multiplication on binomials.

SA7. Simplify rational algebraic expressions with monomial denominators.

SA8. Factor polynomials by removing the greatest common monomial factor.

'*' indicates that the item was not included in the tests.

'+' indicates that only part of the item was in the tests.

APPENDIX E

Content Specification for the
California Assessment Program

SURVEY OF ACADEMIC SKILLS: GRADE 8
MATHEMATICS

I. Numbers

A. Skills/concepts

1. Order relations and classification

    - Identify missing number on a number line.
    - identify relational symbols (<, = ,> ,=, ,).
    - Identify order relations involving whole numbers, decimals, or fractions.
    - Identify integer expressions and integers on a number line.

2. Number theory

    - Read and write whole numbers and decimals.
    - Identify the place value of a given digit in a whole or decimal number.
    - identify numerals in exponential form and expanded notation.
    - Identify odd, even, prime, and composites.
    - Use rules of divisibility.
    - Identify multiples, factors, prime factors, least common multiple (LCM), and greatest common factor (GCF).
    - Approximate the square root of whole numbers.

3. Properties
    - Identify missing expressions illustrating commutative, associative, and distributive properties.
    - Identify missing numerals to illustrate identity or inverse elements.

B. Applications

- Use knowledge of order relations, classification, number theory, and the properties of addition and multiplication to solve problems in the context of real-life situations.

II. Operations

A. Skills/concepts

1. Whole and rational numbers

- Identify symbols to represent addition, subtraction, multiplication, or division.
- Identify missing numerals to indicate understanding of the addition, subtraction, multiplication, and division algorithms.
- Add and subtract rational numbers in fraction and decimal forms.
- Estimate products and quotients of whole numbers and decimals by rounding.

2. Percents

- Identify equivalent fractions, decimals, and percents.
- Identify equal ratios.
- Identify a given percent of a given number.
- Identify the percent a given number is of another given number.
- Identify the number that is a given percent of another given number.

B. Applications

1. One-step

- Use knowledge of operations on whole numbers, fractions, or decimals to solve simple one-step problems in the context of real-life situations.

2. Two or more steps

- Use knowledge of operations on whole numbers, fractions, or decimals to solve two- (or more) step problems in the context of real-life situations.

III. Algebra

A. Skills/concepts

1. Expressions, equations, & inequalities

- Translate simple English phrases and sentences into algebraic expressions, equations, or inequalities and vice versa.
- Evaluate simple expressions involving one or more operations.

- Identify equivalent simplified values of arithmetic

- expressions using the standard order of operations.
- Add, subtract, multiply, and divide integers.
- Solve for unknowns in simple linear equations.

2. Graphs and functions

   - Identify coordinates of points on the coordinate plane.
   - Identify the missing number in a function table of ordered pairs of numbers.
   - Identify the graph of a simple linear equation.]

B. Applications

   - The student will apply his or her knowledge of algebraic expressions, equations, inequalities, coordinate graphs, or functions to solve problems in the context of real-life situations.

IV. Geometry

A. Skills/concepts

1. Geometric terms and figures

   - Identify two- and three-dimensional geometric figures (quadrilateral, parallelogram, rectangle, square, circle, hexagon, triangle, closed figure, cube, prism, cone, sphere).
   - Identify geometric terms (diameter, radius, circumference, arc, chord, tangent, point, midpoint, endpoint, line, ray., line segment, intersection, union, perpendicular, parallel, vertical, diagonal, side, edge, axis, face, region, adjacent, interior).
   - Identify angles (right, acute, straight, obtuse, and adjacent) and triangles.
   - Identify approximate measure in degrees of angles.

2. Relationships

   - Identify parallel or perpendicular lines and planes.
   - Identify figures that are congruent, similar, or have symmetry.
   - Identify spatial transformation -- rotations, reflections (flips), and translations (slides) of shapes and figures.
   - Identify basic geometric postulates (sum of interior angles of a triangle is 180 degrees, sum of interior angles of a quadrilateral is 360 degrees, two points determine a line, three noncolinear points determine a plane).

B. Applications

   - Use knowledge of geometric terms, figures, relationships, or postulates to solve problems.

V. Measurement

A. Skills/concepts

1. Units and estimates

- Estimate measures (linear and other than linear) of familiar objects in U.S. Customary or nonstandard units.
- Estimate measures of familiar objects in the metric system.
- Identify equivalent measures (for length, area, volume, and mass) in U.S. Customary units.
- Identify equivalent measures (for length, area, volume, and mass) in the metric system.
- Calculate the appropriate measure given a basic conversion table.

2. Perimeter, area, and volume

- Identify formulas for perimeter, area, and volume.
- Calculate perimeter, circumference, area and volume of geometric figures.
- Use nonstandard units to measure length, area, and volume of geometric figures.

B. Applications

- Use knowledge of measurement to solve problems.

VI. Probability and Statistics

A. Probability

- Identify the probability of a given event.
- Identify the probability of the complement of an event.
- Identify the probability of an event that is certain to occur or an event that is certain not to occur.
- Identify the probability of a disjoint (independent) event.
- Identify the empirical probability of an event from a sample of observed outcomes.
- Use fundamental counting procedure to determine the number of outcomes in an event.

B. Statistics

- Identify the mean or average of a given list of data.
- Identify the range of a given list of data.
- Identify the median of a given list of data.
- Identify the mode of a given list of data.
- Determine the frequency distribution for a given list of data.

VII. Tables, Graphs, & Integrated Applications

A. Tables and graphs

- Interpret data given in the form of line, bar, circle, and pictographs.

B. Integrated applications

- Interpret information from maps or road signs.
- Interpret information from advertisements or notices.
- Interpret information from commercial charts or tables.
- Interpret information from schedules.

VIII. Problem Solving

A. Formulations of problem

- Identify questions arising from a described practical situation.
- Identify problems arising from a mathematical model (graph, equation, diagram, table, or number line).
- Identify statements that can be made using information given or gathered.
- identify reasonable conclusions drawn from a mathematical model.

B. Analysis of problem

- Identify the facts in a given situation
- Identify (a) appropriate information, (b) extraneous information, or (c) incomplete information in a given problem.
- Clarify ideas within a problem, (a) forming a mental image of what must be done, (b) restating the problem in simpler form, and (c) identifying similarities and differences between two sets of information.
- Identify a problem having the same underlying mathematical processes (same steps in same sequence) as a given problem.

C. Applying strategies

- Use estimation to predict reasonable solutions for a given problem or identify problem or identify problem solving tactics.
- Identify (a) appropriate operations, (b) appropriate number sentence or equation, (c) subproblems, or (d) alternative strategies that will lead to a solution of a given problem.
- Identify (a) appropriate drawings or diagrams, (b) appropriate graphs, (c) appropriate tables or charts,

(d) appropriate "guess-and-check" strategies, (e) appropriate patterns, (f) simplifying strategies, (g) "working-backwards" strategies, or (h) mathematical reasoning that will lead to the solution of a given problem.

D. Reasoning and interpretation

- Recognize a sensible solution to a given problem.
- Verify the accuracy of the problem analysis and the mathematical work by checking a solution in the context of a given problem.
- Identify reasonable conclusions or interpretations from the solution of a problem.
- Identify "simplifying assumptions" that were made in the analysis and solution of a problem.
- Interpret the effect of "simplifying assumptions" that were made in the formation of a mathematical model upon the validity of a solution of a given problem.

E. Solution of problems (aggregations of applications, probability and statistics, tables, graphs, & integrated applications)